Math 142, Final Exam, Fall 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. There are **18** problems on **2** sides.

No Calculators or Cell phones. Write in complete sentences. Explain what you are doing VERY thoroughly.

1. (6 points) Find $\int \sin^5 x dx$. Check your answer.
2. (6 points) Find $\int \sin^4 x dx$.
3. (6 points) Find $\int \tan^5 x dx$. Check your answer.
4. (6 points) Find $\int \frac{1}{2x^2+4x+11} dx$. Check your answer.
5. (6 points) Find $\int \frac{6x^3+8x^2+2x+3}{x^4+x^3} dx$. Check your answer.
6. (6 points) Find $\int \arctan x dx$. Check your answer.
7. (6 points) Give an upper bound on the difference between $\sum_{n=1}^{\infty} \frac{1}{n^3}$ and $\sum_{n=1}^{10} \frac{1}{n^3}$. Be sure to explain what you are doing.
8. (6 points) Compute $\lim_{x\to 0} \frac{\cos(x^2)-1+\frac{x^4}{2}-\frac{x^8}{4!}}{x^{12}}$. Be sure to explain what you are doing.
9. (6 points) Estimate $\int_0^{1/10} \sin(x^3) dx$ with an error of at most 10^{-10}. Be sure to explain what you are doing.
10. (6 points) Let $f(x) = \ln x$. Find the second Taylor polynomial $T_2(x)$ for $f(x)$ about $a = 1$.
11. (5 points) Keep the notation of problem 11. Give an upper bound for the distance between $f(x)$ and $T_2(x)$ for $.9 < x < 1.1$. Be sure to explain what you are doing.
12. (5 points) What familiar function is equal to $x - \frac{x^3}{2} + \frac{x^5}{4!} - \frac{x^7}{6!} + \frac{x^9}{8!} + \ldots$?

PLEASE TURN OVER.

13. (5 points) Where does the power series $f(x) = \sum_{n=1}^{\infty} \frac{(x-5)^n}{n2^n}$ converge? Be sure to explain what you are doing.

14. (5 points) Does the series $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$ converge? Justify your answer very thoroughly.

15. (5 points) Does the series $\sum_{k=1}^{\infty} \frac{\sqrt{k^2-1}}{k^3+2k^2+5}$ converge? Justify your answer very thoroughly.

16. (5 points) Consider the sequence defined by $a_1 = 2$ and $a_{n+1} = \frac{1}{4-a_n}$.
    (a) Prove that $0 < a_n \leq 2$ for all positive integers n.
    (b) Prove that $a_{n+1} \leq a_n$ for all positive integers n.
    (c) State the Completeness Axiom and draw a conclusion about the sequence $\{a_n\}$ from the Completeness Axiom.
    (d) Find the limit of the sequence $\{a_n\}$.

17. (5 points) Consider the region bounded by $y = x^2$ and $x = y - 6$. Revolve the region about $y = -4$. Find the volume of the resulting solid.

18. (5 points) Consider a solid S. The base of S is the triangle with vertices $(0,0)$, $(1,0)$, and $(0,1)$. The cross sections of S perpendicular to the x-axis are squares. Find the volume of S.