Math 142, Final Exam, Fall 2010

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. Each problem is worth 5 points. SHOW your work. $\boxed{CIRCLE}$ your answer.

No Calculators or Cell phones.

1. Consider the region bounded by $y = x^2$ and $x = y - 6$. Revolve the region about $x = -4$. Find the volume of the resulting solid.

2. Consider a solid S. The base of S is an elliptical region with boundary curve $4x^2 + 9y^2 = 36$. The cross sections of S perpendicular to the x-axis are squares. Find the volume of S.

3. Consider the sequence $\{a_n\}$ with $a_0 = 0$, and for all $n \geq 1$, $a_n = \sqrt{20 + a_{n-1}}$. Prove that this sequence is increasing. Prove that this sequence is bounded. Deduce that the sequence converges. Find the limit of the sequence.

4. Suppose that the government pumps an extra \$1 billion into the economy. Assume that each business and individual saves 25% of its income and spends the rest, so that of the initial \$1 billion, 75% is respent by individuals and businesses. Of that amount, 75% is spent, and so forth. What is the total increase in spending due to the government action?

5. Let $f(x)$ be the power series $\sum_{n=0}^{\infty} \frac{(x-3)^n}{n3^n}$. Where does $f(x)$ converge?

6. Find the second Taylor Polynomial $T_2(x)$ for $f(x) = \sqrt{x+1}$ about $a = 0$. Give an upper bound for the error that is introduced if $T_2(x)$ is used to approximate $f(x)$ for $0 \leq x \leq .2$.

7. Find $\lim_{x \to 0} \frac{\sin x^2 - x^2 + \frac{1}{6}x^6}{x^9}$.

8. Approximate $\int_0^1 x \sin(x^3) dx$ with an error at most 10^{-3}. **Justify your answer very thoroughly**.

9. Approximate $\sum_{n=1}^{\infty} \frac{1}{n^4}$ with an error of at most $\frac{1}{100}$. **Explain what you are doing very thoroughly.**

10. Does $\sum_{n=1}^{\infty} \frac{2n+3}{4n^2+5}$ converge? **Justify your answer very thoroughly**.

11. Find $\int_0^3 \frac{1}{(x-1)^2} dx$.

12. Define the definite integral. Give a complete definition. Be sure to explain all of your notation.

13. Find $\int e^{2x} \cos x dx$. **Check your answer.**

14. Find $\int \frac{3x^2-x+3}{(x-2)(x^2+9)} dx$. **Check your answer.**

15. Find $\int \sin^4 x \cos^3 x dx$. **Check your answer.**

16. Find $\int \frac{dx}{x\sqrt{\ln x}}$. **Check your answer.**

17. Find $\int \sec^6 t dt$. **Check your answer.**

18. Find $\int \tan^5 x dx$. **Check your answer.**

19. Find $\int \sqrt{5+4x-x^2} dx$. **Check your answer.**

20. Find $\int \sqrt{x^2+2x} dx$. **Check your answer.**