Math 142, Final Exam, Fall 2009

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: turn the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 200 points. There are **20** problems. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. Find $\int 3\sqrt{4+2x}dx$. **Check your answer.**

2. Define the definite integral. Give a complete definition. Be sure to explain all of your notation.

3. Find the area between $x = y^2$ and $y = x - 2$.

4. Find the volume of the solid whose base in the xy plane is the region bounded between the curve $y = x^3$ and the y-axis from $y = 0$ to $y = 1$ and whose cross sections taken perpendicular to the y-axis are squares.

5. A conical water tank whose base is a circle of radius 10 feet and whose height is 30 ft is filled with water to a depth of 15 feet. How much work is required to pump all of the water out through a hole in the top of the tank? (The density of water is 62.4 pounds per cubic foot.) **Be sure to include units in your answer.**

6. Consider the region in the first quadrant bounded by $y = x$ and $y = x^2$. Rotate this region about the y-axis. Find the volume.

7. Find the length of $y = x^{3/2}$ from $(1,1)$ to $(2, 2\sqrt{2})$.

8. Find $\int xe^x dx$. **Check your answer.**

9. Find $\int \tan^2 x \sec^4 x dx$. **Check your answer.**

10. Find $\int \frac{\sqrt{x^2-25}}{x} dx$. **Check your answer.**

11. Find $\int \frac{x+1}{(x-1)^2} dx$. **Check your answer.**

12. Find $\int_0^{\infty} \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$.

13. Find the limit of the sequence whose n^{th} term is $a_n = \dfrac{\sin(4n)}{n}$.

14. Consider the series $\sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+1}\right)$. Find a closed formula for the n^{th} partial sum $s_n = \sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+1}\right)$. **Be sure to answer the question I have asked!**

15. Consider the series $\sum_{k=1}^{\infty} (23)^k$. Find a closed formula for the n^{th} partial sum $s_n = \sum_{k=1}^{n} (23)^k$. **Be sure to answer the question I have asked!**

16. Does the series $\displaystyle\sum_{k=1}^{\infty} \frac{\sqrt{k}+1}{k^2+2k}$ converge? Justify your answer.

17. Find the third Taylor polynomial for $f(x) = \sqrt{x}$ about $c = 1$.

18. Approximate $\displaystyle\sum_{n=10}^{\infty} \frac{1}{n^2}$. I want you to give both an under estimate and an over estimate. Explain what you are doing.

19. Approximate $e^{\frac{-1}{100}}$ with an error at most 10^{-9} . Explain what you are doing.

20. What is the exact value of the sum $\sum_{k=1}^{\infty} \frac{1}{k(4^k)} = \frac{1}{4} + \frac{1}{2(4^2)} + \frac{1}{3(4^3)} + \frac{1}{4(4^4)} \dots$?