Math 142, Final Exam, Fall 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 100 points. SHOW your work. This work must be coherent and correct. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators or Cell phones.**

1. (9 points) Find $\int \sin^5 x \cos^2 x dx$. **CHECK** your answer.

2. (9 points) Find $\int \frac{1}{2x^2+4x+11} dx$. **CHECK** your answer.

3. (9 points) Find $\int \frac{x^2+3}{(x+1)(x^2+1)} dx$. **CHECK** your answer.

4. (9 points) Find the volume of the solid that is obtained by revolving the region between $y = x-2$ and $x = y^2$ about the line $y = 5$. **There is no need to do the integral:** set the integral up and stop. **You must draw a meaningful picture.**

5. (8 points) Consider a solid S . The base of S is the region in the xy plane between $y = x - 2$ and $x = y^2$. (This is the same region as in problem 4.) Each cross section of S parallel to the xz plane is a square. Find the volume of S . **There is no need to do the integral:** set the integral up and stop. **You must draw a meaningful picture.**

6. (8 points) Find the second Taylor polynomial $T_2(x)$ for $f(x) = x^{5/3}$ centered at $x = 8$. **Justify your answer very thoroughly. Write in complete sentences.**

7. (8 points) Keep the situation of problem 6. Give an upper bound for the distance between $f(x)$ and $T_2(x)$ for $|x-8| \leq 1$. **Justify your answer very thoroughly. Write in complete sentences.**

PLEASE TURN OVER.

8. (8 points) What function that one might consider in Calculus is equal to

$$f(x) = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} + \frac{x^{11}}{11 \cdot 5!} - \frac{x^{15}}{15 \cdot 7!} + \frac{x^{19}}{19 \cdot 9!} - \frac{x^{23}}{23 \cdot 11!} + \dots ?$$

Justify your answer very thoroughly. Write in complete sentences.

9. (8 points) Does $\sum_{n=1}^{\infty} (2^{1/n} - 1)$ converge? **Justify your answer very thoroughly. Write in complete sentences.**

10. (8 points) Find $\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2}}{x^5}$. **Justify your answer very thoroughly. Write in complete sentences.**

11. (8 points) Does $\sum_{n=1}^{\infty} \frac{\sin(1/n)}{\sqrt{n}}$ converge? **Justify your answer very thoroughly. Write in complete sentences.**

12. (8 points) Where does $f(x) = \sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n \sqrt{n}}$ converge? **Justify your answer very thoroughly. Write in complete sentences. Be sure to check "the endpoints" carefully, if there are endpoints.**