Math 142, Final Exam , Spring 2006

There are 20 problems. Each problem is worth 10 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website a few hours after the exam is finished.

1. Find $\int \frac{e^x}{\sqrt{e^x+1}}\,dx$. **Check your answer.**

2. Find $\int \sin^3 x \cos^2 x\,dx$. **Check your answer.**

3. Find $\int \sqrt{x^2+1}\,dx$. **Check your answer.**

4. Find $\int \ln x\,dx$. **Check your answer.**

5. Find $\int \frac{4x^3+6x^2+x+2}{x^4+x^2}dx$. **Check your answer.**

6. Find $\lim_{x\to\infty}\left(\frac{x}{x-4}\right)^x$.

7. Find the area between $y=x$ and $x+y^2=2$.

8. Consider the sequence $\{a_n\}$ with $a_1=10$, and $a_n=\frac{1}{2}[a_{n-1}+\frac{3}{a_{n-1}}]$ for $n\geq 2$. Prove that the sequence $\{a_n\}$ converges. Find the limit of the sequence $\{a_n\}$.

9. A conical water tank sits with its base on the ground. The radius of the base is 10 feet. The height of the tank is 30 feet. The tank is filled to a depth of 25 feet. How much work is required to pump all of the water out through a hole in the top of the tank? The density of water is 62.4 lb/ft ^{3} . Be sure to give the units for your answer.

10. Consider the region in the first quadrant which is bounded by $y=x^2$, the x-axis, and $x=1$. Revolve this region about the line $x=5$. What is the volume of the resulting solid?

11. Find the length of $24xy = y^4 + 48$ from $y = 2$ to $y = 4$.

12. Let $f(x) = \sum_{k=1}^{\infty} \frac{(x-2)^k}{5^k k}$. Find all real numbers x for which $f(x)$ converges. **Justify your answer.**

13. Find $\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2} - \frac{x^6}{3!}}{x^8}$. **Justify your answer.**

14. Does $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{k^3 + 1}$ converge? **Justify your answer.**

15. Does $\sum_{k=1}^{\infty} \frac{5^k + k}{k! + 3}$ converge? **Justify your answer.**

16. What is the exact sum of the series $\sum_{k=0}^{\infty} \frac{k}{3^k}$? **Justify your answer.**

17. Approximate $\sum_{k=1}^{\infty} \frac{1}{k^4}$ with an error at most $\frac{1}{1000}$. **Justify your answer.**

18. Approximate $\int_0^{\frac{1}{10}} \sin(x^2)\, dx$ with an error at most $\frac{1}{1000}$. **Justify your answer.**

19. Find the Taylor polynomial $P_3(x)$ of order 3 for the function $f(x) = \ln x$ about $a = 1$.

20. Keep the notation of problem 19. Find an upper bound for the error that is introduced if $P_3(x)$ is used to approximate $f(x)$ when $|x - 1| < .1$