Math 142   Final Exam   Fall 2004

PRINT Your Name:__________________________________

There are 20 problems on 10 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS! CHECK** your answer whenever possible.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**. Otherwise, get your grade from VIP.

I will post the solutions on my website when the exam is finished.

1. Find $\frac{d}{dx}\left(xe^{x^2}\right)$.

2. Find $\int xe^{x^2}\,dx$. Check your answer.

3. Simplify $\sin(2\arccos(\frac{3}{4}))$.

4. Find $\int \sin^4 x\cos^3 x\,dx$. Check your answer.

5. Find $\int \sin^2 x\,dx$.

6. Find $\int \frac{x}{x^2+4}\,dx$. Check your answer.

7. Find $\int \frac{1}{x^2+4}\,dx$. Check your answer.

8. Find $\int \frac{1}{\sqrt{x^2+4}}\,dx$. Check your answer.

9. Let $f(x) = x\ln x$. What is the domain of $f(x)$? Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find the local maxima, local minima, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

10. Find $\lim\limits_{x\to 0} \dfrac{e^{x^2} - 1 - x^2 - \frac{x^4}{2} - \frac{x^6}{6} - \frac{x^8}{24}}{x^{10}}$. Justify your answer.

11. Find $\int \frac{x\,dx}{(x-3)^2}$. Check your answer.

12. Find $\int \ln x\,dx$. Check your answer.

13. Find $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$.

14. Find the limit of the sequence whose n^{th} term is $a_n = \left(\frac{n-1}{n}\right)^n$. Justify your answer.

15. What familiar function is equal to

$$f(x) = 1 + 2x + 3x^2 + 4x^3 + 5x^4 + \ldots ?$$

Justify your answer.

16. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n+3}{n^2\sqrt{n}}$ converge or diverge? Justify your answer.

17. Does the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{n!}$ converge or diverge? Justify your answer.

18. Find the Taylor polynomial $P_3(x)$ for $f(x) = (1+x)^{3/2}$ about $a = 0$ and bound the error $R_3(x)$ if $-.1 \le x \le 0$.

19. Use the Parabolic Rule to approximate the amount of water required to fill a pool shaped like the picture below to a depth of 6 feet. (See a different page.) All dimensions are in feet. Recall that Parabolic Rule says that if n is even, then $\int_a^b f(x)dx$ is equal to

$$\frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right] + E_n,$$

for $h = \frac{b-a}{n}$, $x_i = a + hi$, and $E_n = -\frac{(b-a)^5}{180n^4} f^{(4)}(c)$ for some c with $a \le c \le b$. (Just record the sum. You are not required to perform any addition or multiplication.)

20. Carbon 14, an isotope of carbon, is radioactive and decays at a rate proportional to the amount present. Its half life is 5730 years; that is, it takes 5730 years for a given amount of carbon 14 to decay to one-half its original size. If 10 grams was present originally, how much will be left after 2000 years? (You may leave ln in your answer.)