Math 142, Exam 4, Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 100 points. There are 10 problems. Each problem is worth 10 points.

SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators or Cell phones.**

I will post the solutions on my website sometime this afternoon.

If I know your e-mail address, I will e-mail your grade to you as soon as the exam is graded. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. Write $12.457575757\ldots$ as a quotient of two integers.

2. Approximate $\sum\limits_{k=1}^{\infty} \frac{1}{k^4}$ with an error at most $\frac{1}{2000}$. Explain **very thoroughly**.

3. Give a closed formula for $3^2 + 3^3 + 3^4 + \cdots + 3^{99}$. There should be no dots or summation signs in your answer. Your answer should be exactly equal to the given number.

4. Does $\sum\limits_{k=1}^{\infty} \frac{4+\cos^2 k}{k^3}$ converge? Explain **very thoroughly**.

5. Does $\sum\limits_{k=1}^{\infty} \frac{k^2}{2^k}$ converge? Explain **very thoroughly**.

6. Does $\sum\limits_{k=1}^{\infty} \frac{2}{k+5}$ converge? Explain **very thoroughly**.

7. Does $\sum\limits_{k=1}^{\infty} \left(\frac{k-5}{k}\right)^k$ converge? Explain **very thoroughly**.

8. Consider the sequence $\{a_n\}$ with $a_1 = \sqrt{30}$, and $a_n = \sqrt{30 + a_{n-1}}$ for $n \geq 2$. Prove that the sequence $\{a_n\}$ is increasing. Identify an upper bound for the sequence $\{a_n\}$. Prove that your candidate for an upper bound really is an upper bound. Find the limit of the sequence $\{a_n\}$. Explain **very thoroughly**.

9. Find a closed formula for the n^{th} partial sum of the series

$$\ln\left(1 - \frac{1}{4}\right) + \ln\left(1 - \frac{1}{9}\right) + \ln\left(1 - \frac{1}{16}\right) + \ldots .$$

Explain **very thoroughly**. Be sure to do the problem that I picked – and not some other problem.

10. Find all values of x for which the series:

$$\frac{1}{x^2} + \frac{2}{x^3} + \frac{4}{x^4} + \frac{8}{x^5} + \frac{16}{x^6} + \ldots$$

converges. What is the sum of the series? Explain **very thoroughly**.