Math 142, Exam 3, Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 100 points. There are 10 problems. Each problem is worth 10 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators or Cell phones.**

I will post the solutions on my website sometime this afternoon.

If I know your e-mail address, I will e-mail your grade to you as soon as the exam is graded. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. A cone shaped water reservoir is 20 feet in **diameter** across the top and 15 feet deep. If the reservoir is filled to a depth of 10 feet, how much work is required to pump all the water to the top of the reservoir? The density of water is $62.4\,\mathrm{lb/ft}^3$. (There is no need for you to do arithmetic. Leave your answer in terms of sums and products of numbers.)

2. Find $\displaystyle\int_0^5 \frac{1}{(x-4)^2}\,dx$.

3. Find $\displaystyle\int \sin^3 x\,dx$. **Check your answer.**

4. Find $\displaystyle\int \sin^2 x\,dx$.

5. Find $\displaystyle\int \sec^3 x\,dx$. **Check your answer.**

6. Find $\displaystyle\int \frac{-2x^2-2x+1}{x^2(x^2+1)}\,dx$. **Check your answer.**

7. What is the limit of the sequence whose n^{th} term is $a_n = \left(\frac{n+5}{n}\right)^n$?

8. Does the integral $\int_2^\infty \frac{x}{x^6+1}\,dx$ converge? Explain your answer **very thoroughly.**

9. Consider the sequence

$$
\begin{aligned}
a_1 &= \sqrt{6} \\
a_2 &= \sqrt{6+\sqrt{6}} \\
a_3 &= \sqrt{6+\sqrt{6+\sqrt{6}}} \\
a_4 &= \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}} \\
&\vdots
\end{aligned}
$$

Assume that you know that the sequence converges. Find the limit of the sequence. Explain your work **very thoroughly.**

10. Trapezoidal rule says that $\int_a^b f(x)dx = T_n + E_n$. The formula for T_n is not needed in this problem. The value of E_n is $E_n = \frac{-(b-a)^3}{12n^2} f''(c)$ for some c between a and b . Suppose that Trapezoidal rule is used to approximate $\int_1^3 \frac{1}{x}\,dx$, with $n = 10$. Find a decent upper bound for $|E_n|$. Explain your work **very thoroughly**.