PRINT Your Name:____________________________________

Math 142 Exam 4 Fall 2004

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

If you would like, I will leave your exam outside my office after I have graded it. (I will send you an e-mail when I am finished with it.) You may pick it up any time between then and the next class. **Let me know if you are interested.**

I will post the solutions on my website at about 6:00 PM today.

1. What is the limit of the sequence:

$$\sin 1,\ 2\sin\frac{1}{2},\ 3\sin\frac{1}{3},\ 4\sin\frac{1}{4},\ 5\sin\frac{1}{5},\ \ldots?$$

   Explain your answer.

2. Does the series $\displaystyle\sum_{n=2}^{\infty}\frac{3^n}{4^{n+1}}$ converge? Explain your answer.

3. Does the series $\displaystyle\sum_{n=2}^{\infty}\frac{n}{n^2-1}$ converge? Explain your answer.

4. Does the series $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$ converge? Explain your answer.

5. Does the series

$$1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\ldots$$

   converge? Explain your answer.

6. For which values of x does the power series $f(x)=\displaystyle\sum_{n=1}^{\infty}\frac{(x-3)^n}{n2^n}$ converge? Explain your answer.

7. Approximate $\displaystyle\int_0^1 \sin(x^2)\,dx$ with an error of at most $\frac{1}{10^4}$. Explain your answer.

8. What familiar series is equal to

$$x^2+\frac{x^4}{2!}+\frac{x^6}{3!}+\frac{x^8}{4!}+\frac{x^{10}}{5!}+\ldots?$$

   Explain your answer.

9. Find the Taylor Polynomial $P_3(x)$ for the function $f(x) = \sqrt{x}$ about $a = 1$.

10. Give an upper bound for the difference between $\sum_{n=1}^{10} \frac{1}{n^4}$ and $\sum_{n=1}^{\infty} \frac{1}{n^4}$. I expect your upper bound to be relatively small and correct. Be sure to explain what you are doing and why you are allowed to do it.