Math 142, Exam 4, Fall 2010

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer.
No Calculators or Cell phones.

1. (6 points) Consider the region bounded by $x = y^2$ and $y = x - 6$. Revolve the region about $y = 4$. Find the volume of the resulting solid.

2. (6 points) Consider a solid S. The base of S is an elliptical region with boundary curve $9x^2 + 4y^2 = 36$. The cross sections of S perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Find the volume of S.

3. (6 points) Consider the sequence $\{a_n\}$ with $a_0 = 0$, and for all $n \geq 1$, $a_n = \sqrt{6 + a_{n-1}}$. Prove that this sequence is increasing. Prove that this sequence is bounded. Deduce that the sequence converges. Find the limit of the sequence.

4. (6 points) A ball is dropped from a height of 100 feet. Each time it hits the ground, it rebounds to $\frac{2}{3}$ its previous height. Find the total distance the ball travels before comming to rest.

5. (6 points) Let $f(x)$ be the power series $\sum\limits_{n=0}^{\infty} \frac{(x-3)^n}{2n+1}$. Where does $f(x)$ converge?

6. (6 points) Find the second Taylor Polynomial $T_2(x)$ for $f(x) = \sqrt{x}$ about $a = 4$. Give an upper bound for the error that is introduced if $T_2(x)$ is used to approximate $f(x)$ for $4 \leq x \leq 4.2$.

7. (7 points) Find $\lim\limits_{x \to 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$.

8. (7 points) Approximate $\int_0^1 x \cos(x^3) dx$ with an error at most 10^{-3}. **Justify your answer very thoroughly**.