Math 142, Exam 4, Spring 2006

There are 10 problems. Each problem is worth 10 points. Write in complete sentences. JUSTIFY EVERY ASNSWER **VERY THOROUGHLY**.

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website a few hours after the exam is finished.

1. Does the series $\sum_{k=1}^{\infty} \frac{1}{2+\frac{1}{k}}$ converge? **Justify your answer.**

2. Does the series $\sum_{k=1}^{\infty} \frac{k}{2^k}$ converge? **Justify your answer.**

3. Does the series $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}$ converge? **Justify your answer.**

4. Does the series $\sum_{k=1}^{\infty} \frac{\arctan k}{k^2}$ converge? **Justify your answer.**

5. Does the series $\sum_{k=2}^{\infty} \frac{1}{\sqrt{k(k-1)}}$ converge? **Justify your answer.**

6. Consider the sequence $\{a_n\}$ with $a_1 = \sqrt{20}$, and $a_n = \sqrt{20 + a_{n-1}}$ for $n \ge 2$. Prove that the sequence $\{a_n\}$ converges. Find the limit of the sequence $\{a_n\}$.

7. Approximate the sum $\sum_{k=1}^{\infty} \frac{1}{k^3}$ with an error of at most $\frac{1}{100}$. **Justify your answer.**

8. Approximate the sum $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^3}$ with an error of at most $\frac{1}{100}$. **Justify your answer.**

9. A ball is dropped from the height of 40 feet. Each time it hits the floor it rebounds to $\frac{5}{7}$ its previous height. Find the total distance it travels. Explain what you are doing.

10. Give a closed formula for $s_n = \sum_{k=2}^{n} \ln\left(1 - \frac{1}{k^2}\right)$. (Your formula should be exactly equal to the sum I have given. Your formula should not contain any dots or any summation signs.) Explain what you are doing.