Math 142, Exam 3, Spring 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones. I will post the solutions on my website.

1. (7 points) Find the volume of the solid obtained by rotating the region bounded by $y = x^2$, the x-axis, $x = 1$, and $x = 2$ about the line $x = 3$.

2. (7 points) Does the series $\sum_{n=2}^{\infty} \frac{1+\sin n}{10^n}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

3. (7 points) Does the series $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{n-1}$ converge? **Justify your answer very thoroughly. Use complete sentences.**

4. (7 points) Does the series $\sum_{n=2}^{\infty} (1 - \frac{1}{n})^n$ converge? **Justify your answer very thoroughly. Use complete sentences.**

5. (7 points) Estimate the distance between $\sum_{n=2}^{10} (\frac{1}{n^4})$ and $\sum_{n=2}^{\infty} (\frac{1}{n^4})$. I want your estimate to be close to, but larger than the exact distance. **Justify your answer very thoroughly. Use complete sentences.**

6. (7 points) Express $3.4174174174\ldots$ as a ratio of integers. **Justify your answer very thoroughly. Use complete sentences.**

7. (8 points) **Justify your answer very thoroughly. Use complete sentences.** Consider the sequence $\{a_n\}$ with $a_1 = 0$ and $a_n = \sqrt{20 + a_{n-1}}$ for $n \geq 2$.
   (a) Show that $0 \leq a_n \leq 10$ for all n.
   (b) Show that $\{a_n\}$ is an increasing sequence.
   (c) Explain why the sequence $\{a_n\}$ converges.
   (d) Find the limit of the sequence $\{a_n\}$.