## Math 142, Exam 3, Spring 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are 7 problems. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

**No Calculators or Cell phones.**

1. (7 points) Consider the solid whose base is bounded by $\frac{x^2}{9}+\frac{y^2}{16}=1$ in the $xy$-plane. Each cross section of the solid perpendicular to the $x$-axis and perpendicular to the base is an equilateral triangle. Find the volume of the solid. **You must draw a meaningful picture.**

2. (8 points) Consider the sequence defined by $a_1=1$ and $a_{n+1}=\frac{1}{3}(a_n+4)$, for $1\le n$.
   (a) Prove that the sequence $\{a_n\}$ is increasing. **Justify your answer thoroughly. Write in complete sentences.**
   (b) Prove that the sequence $\{a_n\}$ is bounded. **Justify your answer thoroughly. Write in complete sentences.**
   (c) State the Completeness Axiom and draw a conclusion about the sequence $\{a_n\}$ from the Completeness Axiom. **Justify your answer thoroughly. Write in complete sentences.**
   (d) Find the limit of the sequence $\{a_n\}$. **Justify your answer thoroughly. Write in complete sentences.**

3. (7 points) **Justify your answer thoroughly. Write in complete sentences.**
   (a) Find a closed formula for the partial sum $s_N=\sum\limits_{k=1}^{N}\left(\frac{1}{k}-\frac{1}{k+3}\right)$, provided $N$ is big.
   (b) What is the sum of the series $\sum\limits_{k=1}^{\infty}\left(\frac{1}{k}-\frac{1}{k+3}\right)$?

4. (7 points) Find a closed formula for the sum $S=\sum\limits_{k=3}^{100}2^k$. **Justify your answer thoroughly. Write in complete sentences.** Answer the question that I asked, not some other question.

**Please turn over.**

5. (7 points) Where does the power series $f(x) = \sum_{n=1}^{\infty} \frac{(x-9)^n}{n^2 3^n}$ converge? **Justify your answer thoroughly. Write in complete sentences.**

6. (7 points) Does $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln x}}$ converge? **Justify your answer thoroughly. Write in complete sentences.**

7. (7 points) Does $\sum_{n=2}^{\infty} (1 - \frac{1}{n})^n$ converge? **Justify your answer thoroughly. Write in complete sentences.**