Math 142, Exam 3, Fall 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **7** problems on **2** sides.

No Calculators or Cell phones. Write in complete sentences. Explain what you are doing VERY thoroughly.

1. (8 points) Consider the sequence defined by $a_1 = 2$ and $a_{n+1} = \frac{1}{4-a_n}$.
   (a) Prove that $0 < a_n \leq 2$ for all positive integers n.
   (b) Prove that $a_{n+1} \leq a_n$ for all positive integers n.
   (c) State the Completeness Axiom and draw a conclusion about the sequence $\{a_n\}$ from the Completeness Axiom.
   (d) Find the limit of the sequence $\{a_n\}$.

2. (7 points) Find the limit of the sequence whose n^{th} term is $a_n = (\frac{n-3}{n})^{2n}$.

3. (7 points) Consider the series $\sum_{k=3}^{\infty} 6(\frac{1}{3})^k$. Does the series converge? Find the sum of the series if possible. Explain what you are doing in great detail.

4. (7 points) Consider the series $\sum_{k=2}^{\infty} (\frac{1}{k} - \frac{1}{k+2})$. For each integer n, with $2 \leq n$, let $s_n = \sum_{k=2}^{n} (\frac{1}{k} - \frac{1}{k+2})$
   (a) Write down s_5. Be sure to cancel everything that cancels.
   (b) Find a closed formula for s_n. Recall that a closed formula does not have any summation signs or any dots.
   (c) Find $\lim_{n\to\infty} s_n$.
   (d) Does the series $\sum_{k=2}^{\infty} (\frac{1}{k} - \frac{1}{k+2})$ converge?
   (e) Find the sum of the series $\sum_{k=2}^{\infty} (\frac{1}{k} - \frac{1}{k+2})$, if possible.

5. (7 points) Estimate $\sum_{k=1}^{\infty} \frac{1}{k^5}$ with an error at most $\frac{1}{1000}$.

PLEASE TURN OVER.

6. (7 points) Does the series $\sum_{k=1}^{\infty} \frac{1}{k-\ln k}$ converge? **Justify your answer VERY thoroughly.**

7. (7 points) Does the series $\sum_{k=1}^{\infty} \frac{k}{2k+3}$ converge? **Justify your answer VERY thoroughly.**