

Math 142, Exam 3, Fall 2010

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. **SHOW** your work. *CIRCLE* your answer. **CHECK** your answer whenever possible.

No Calculators or Cell phones.

1. (6 points) Consider the region bounded by $x = y^2$ and $y = x - 6$. Revolve the region about $x = 10$. Find the volume of the resulting solid.
2. (6 points) Consider a wedge cut from a cylinder of radius 4. This wedge is cut using 2 planes. The first plane is perpendicular to the axis of the cylinder. The second plane intersects the first plane through a diameter of the cylinder. The angle of intersection of the two planes is 45 degrees. Find the volume of the wedge.
3. (6 points) Consider the sequence $\{a_n\}$ with $a_0 = 1$, and for all $n \geq 1$, $a_n = \sqrt{2a_{n-1}}$. Prove that this sequence is increasing. Prove that this sequence is bounded. Deduce that the sequence converges. Find the limit of the sequence.
4. (6 points) Express the repeating decimal $r = 1.53\overline{42}$ as the ratio of two integers. **Explain what you are doing very thoroughly.**
5. (6 points) Approximate $\sum_{n=1}^{\infty} \frac{1}{n^3}$ with an error of at most $\frac{1}{100}$. **Explain what you are doing very thoroughly.**
6. (6 points) Approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^3}$ with an error of at most $\frac{1}{100}$. **Explain what you are doing very thoroughly.**
7. (7 points) Does $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ converge? **Justify your answer very thoroughly.**
8. (7 points) Does $\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$ converge? **Justify your answer very thoroughly.**