Math 142, Exam 3, Fall 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. SHOW your work. This work must be coherent and correct. $\boxed{CIRCLE}$ your answer. **No Calculators or Cell phones.**

The solutions will be posted later today.

1. (8 points) Find the volume of the solid that is obtained by revolving the region bounded by $y^2 = x$ and $x - y = 2$ about the line $y = -6$. **You must draw a meaningful picture.** (There is no need for you to do the final arithmetic. That is, you may stop as soon as you have plugged the endpoints into an anti-derivative.)

2. (7 points) Consider a solid S whose base in the xy plane is the region bounded by $y^2 = x$ and $x - y = 2$. Each cross-section of S perpendicular to the y-axis is a square. Find the volume of S. **You must draw a meaningful picture.** (There is no need for you to do the final arithmetic. That is, you may stop as soon as you have plugged the endpoints into an anti-derivative.)

3. (7 points) Consider the sequence defined by $a_1 = 2$ and $a_{n+1} = \frac{1}{4-a_n}$. **Justify your answers very thoroughly. Write in complete sentences.**
   (a) Prove that $0 < a_n \le 2$ for all positive integers n.
   (b) Prove that $a_{n+1} \le a_n$ for all positive integers n.
   (c) State the Completeness Axiom and draw a conclusion about the sequence $\{a_n\}$ from the Completeness Axiom.
   (d) Find the limit of the sequence $\{a_n\}$.

4. (7 points) Estimate the distance between $\sum\limits_{k=1}^{100} \frac{1}{k^4}$ and $\sum\limits_{k=1}^{\infty} \frac{1}{k^4}$. Your answer should be in the form "The distance between $\sum\limits_{k=1}^{100} \frac{1}{k^4}$ and $\sum\limits_{k=1}^{\infty} \frac{1}{k^4}$ is less than xxx", where xxx is some small positive number that you have calculated. **Justify your answer very thoroughly. Write in complete sentences. You must draw a meaningful picture.**

Please turn over.

5. (7 points) Consider the series $\frac{2}{5} - (\frac{2}{5})^2 + (\frac{2}{5})^3 - (\frac{2}{5})^4 + \ldots$. **Justify your answer very thoroughly. Write in complete sentences.**
   (a) Find a closed formula for the n^{th} partial sum

$$s_n = \tfrac{2}{5} - (\tfrac{2}{5})^2 + (\tfrac{2}{5})^3 - (\tfrac{2}{5})^4 + \cdots + (-1)^{n+1}(\tfrac{2}{5})^n$$

   of this series.
   (b) Find the sum of the entire series.

6. (7 points) Consider the series $\ln \frac{2}{3} + \ln \frac{3}{4} + \ln \frac{4}{5} + \ln \frac{5}{6} \ldots$. **Justify your answer very thoroughly. Write in complete sentences.**
   (a) Find a closed formula for the n^{th} partial sum

$$s_n = \ln \tfrac{2}{3} + \ln \tfrac{3}{4} + \ln \tfrac{4}{5} + \ln \tfrac{5}{6} \cdots + \ln \tfrac{n+1}{n+2}$$

   of this series.
   (b) Find the sum of the entire series.

7. (7 points) Does the series $\sum_{k=1}^{\infty} \frac{2k}{k^2+1}$ converge? **Justify your answer very thoroughly. Write in complete sentences.**