PRINT Your Name:_____

<div align="center">**Quiz for August 25, 2005**</div>

Find an equation for the family of lines tangent to the circle with center at the origin and radius 3.

ANSWER: For each point P on the circle, there is exactly one line in our family! One easy way to think about P is in terms of the angle θ that the line segment from the origin to P makes with the positive x-axis. So, $P = (3\cos\theta, 3\sin\theta)$. The line segement from the origin to P has slope $\frac{3\sin\theta}{3\cos\theta} = \frac{\sin\theta}{\cos\theta}$. So the line from our family through P has slope $-\frac{\cos\theta}{\sin\theta}$ (provided $\sin\theta \neq 0$). This line is $y - 3\sin\theta = -\frac{\cos\theta}{\sin\theta}(x - 3\cos\theta)$. Multiply both sides by $\sin\theta$ to get

$$\sin\theta\, y - 3\sin^2\theta = -\cos\theta\, x + 3\cos^2\theta.$$

This is the same as:

$$\boxed{\sin\theta\, y + \cos\theta\, x = 3.}$$

By the way, the derivation does not make sense for $P = (3, 0)$ or $(-3, 0)$ because in these cases $\sin\theta = 0$; hence, I may not divide by $\sin\theta$; however, my ultimate answer magically works at these points.

Check a few. When $\theta = 0$, the line is $x = 3$, and this is what we expected. When $\theta = \frac{\pi}{4}$, the line is $x + y = 3\sqrt{2}$, and this is also what we expect! When $\theta = \pi/2$, the line is $y = 3$, which is also correct!