PRINT Your Name:____________________

**Quiz for November 8, 2005**

A rectangular area of 3200 $\text{ft}^2$ is to be fenced off. Two opposite sides will use fencing costing \$1 per foot and the remaining sides will use fencing costing \$2 per foot. Find the dimensions of the rectangle of least cost.

**ANSWER:** Let the rectangle be $x$ feet by $y$ feet, where the sides that are $x$ feet long cost \$1 per foot and the sides that are $y$ feet long cost \$2 per foot.

So the total cost is $C = 2x + 4y$. We are told that the area of the rectangle is 3200 $\text{ft}^2$, so $xy = 3200$ or $y = 3200/x$. We must minimize the function $C(x) = 2x + \frac{4(3200)}{x}$. The function is defined for $0 < x$. The domain is not a closed interval, but we are happy to see that $C$ goes to infinity as $x \to 0$, also as $x \to \infty$; thus, the minimum cost occurs some place where $C'(x) = 0$. At any rate

$$C'(x) = 2 - \frac{4(3200)}{x^2} = \frac{2(x^2 - 6400)}{x^2} = \frac{2(x-80)(x+80)}{x^2}.$$

Observe that $x = 80$ is the only value for $x$ which is in the domain of $C(x)$ and also has $C'(x) = 0$. Indeed, we see that the graph of $C(x) = 2x + \frac{4(3200)}{x}$ looks like:

We conclude that

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| the rectangle of least cost is 80 feet by 40 feet, where the fencing that costs \$1 per foot is used on the sides of length 80 feet. |