PRINT Your Name:____________________

Quiz for November 15, 2005

Suppose that f is a differentiable function defined on the interval I and $f'(x) \neq 0$ on I. Prove that the equation $f(x) = 0$ can have at most one real root in I.

ANSWER: We suppose that f has at least two roots in I and we show that this supposition leads to a contradiction. If $a < b$ are in I with $f(a) = f(b) = 0$, then the Mean Value Theorem guarantees that there exists a number c with $a < c < b$ and $f'(c) = \frac{f(b)-f(a)}{b-a} = 0$. However, c is necessarily in I and the hypothesis said that f' is never zero on I. We conclude that it is impossible for f to have at least two roots in I; that is, f has at most one root in I.