PRINT your name ____

Quiz for February 10, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.
The quiz is worth 5 points.

Find the values of the constants k and m, if possible, that will make the function

$$f(x) = \begin{cases} x^2+5 & \text{if } 2 < x \\ m(x+1)+k & \text{if } -1 < x \le 2 \\ 2x^3+x+7 & \text{if } x \le -1 \end{cases}$$

continuous everywhere.

Answer: We see that

$$\lim_{x\to 2^+} f(x) = 9, \quad \lim_{x\to 2^-} f(x) = 3m+k, \quad \lim_{x\to -1^+} f(x) = k, \quad \lim_{x\to -1^-} f(x) = 4.$$

In order for $f(x)$ to be continuous at $x = 2$, we need

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x).$$

In other words, we must have

$$9 = 3m + k.$$

In order for $f(x)$ to be continuous at $x = -1$, we need

$$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^-} f(x).$$

In other words, we must have

(*) $$k = 4.$$

Solve the equations (*) and (**) simultaneously to get $\boxed{k = 4 \ \& \ m = 5/3}$. Be sure to notice that our choice of k and m yields $f(2) = \lim\limits_{x\to 2} f(x) = 9$ and $f(-1) = \lim\limits_{x\to -1} f(x) = 4$.