

PRINT your name _____

Quiz for March 31, 2009 – 9:30 section

Remove everything from your desk except this page and a pencil or pen.

Circle your answer. Show your work.

The quiz is worth 5 points.

Let $f(x) = x \ln x$. What is the domain of $f(x)$? Find all vertical and horizontal asymptotes of $y = f(x)$. Where is $f(x)$ increasing and decreasing? Where is $f(x)$ concave up and concave down? Find the local extreme points and points of inflection of $y = f(x)$? Graph $y = f(x)$.

Answer: The domain of $f(x)$ is the set of all x with $0 < x$. It is easy to see that $\lim_{x \rightarrow \infty} f(x) = +\infty$. One uses L'hospital's rule to see that

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x \ln x = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} = \lim_{x \rightarrow 0^+} -x = 0.$$

We conclude that the graph has no asymptotes. We compute

$$f'(x) = x(1/x) + \ln x = 1 + \ln x.$$

Thus, $f'(x) = 0$ for $x = e^{-1}$; $f'(x)$ is positive for $e^{-1} < x$ and $f'(x)$ is negative for $0 < x < e^{-1}$. Observe that $f(e^{-1}) = -e^{-1}$. Thus,

$f(x)$ is increasing for $e^{-1} < x$;
 $f(x)$ is decreasing for $x < e^{-1}$ and
 $(e^{-1}, -e^{-1})$ is the local minimum of $f(x)$.

We compute $f''(x) = 1/x$ which is always positive. We conclude that the graph is

always increasing and there aren't any points of inflection.

