PRINT your name ______________________________

Quiz for March 31, 2009 – 8:00 section

Remove everything from your desk except this page and a pencil or pen.
$\boxed{\text{Circle}}$ your answer. Show your work.
The quiz is worth 5 points.

Let $f(x) = x \ln x$. What is the domain of $f(x)$? Find all vertical and horizontal asymptotes of $y = f(x)$. Where is $f(x)$ increasing and decreasing? Where is $f(x)$ concave up and concave down? Find the local extreme points and points of inflection of $y = f(x)$? Graph $y = f(x)$.

Answer: $\boxed{\text{The domain of } f(x) \text{ is the set of all } x \text{ with } 0 < x.}$ It is easy to see that $\lim\limits_{x\to\infty} f(x) = +\infty$. One uses L'hopital's rule to see that

$$\lim_{x\to 0+} f(x) = \lim_{x\to 0+} x \ln x = \lim_{x\to 0+} \frac{\ln x}{1/x} = \lim_{x\to 0+} \frac{1/x}{-1/x^2} = \lim_{x\to 0+} -x = 0.$$

We conclude that the graph $\boxed{\text{has no asymptotes}}$. We compute

$$f'(x) = x(1/x) + \ln x = 1 + \ln x.$$

Thus, $f'(x) = 0$ for $x = e^{-1}$; $f'(x)$ is positive for $e^{-1} < x$ and $f'(x)$ is negative for $0 < x < e^{-1}$. Observe that $f(e^{-1}) = -e^{-1}$. Thus,

$$\boxed{\begin{array}{l} f(x) \text{ is increasing for } e^{-1} < x; \\ f(x) \text{ is decreasing for } x < e^{-1} \text{ and} \\ (e^{-1}, -e^{-1}) \text{ is the local minimum of } f(x). \end{array}}$$

We compute $f''(x) = 1/x$ which is always positive. We conclude that the graph is

$\boxed{\text{always increasing and there aren't any points of inflection.}}$