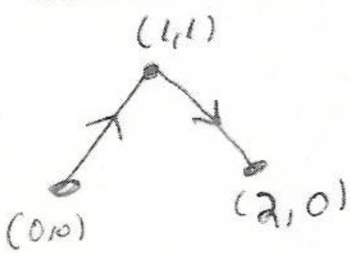
...on Time _____ PRINT your name ____________________

Math 141, Exam 2, Spring 2009

The exam is worth a total of 50 points. There are 11 questions on 5 pages. **SHOW your work. Make your work be coherent and clear.** Write in complete sentences whenever this is possible. CIRCLE your answer. **CHECK** your answer whenever possible. **No Calculators.**

I will post the solutions on my website a few hours after the exam is finished.

1. (5 points) **Parameterize the curve pictured below. Use t as your parameter with $0 \le t \le 2$. The point that corresponds to $t = 0$ is $(0,0)$. The point that corresponds to $t = 1$ is $(1,1)$. The point that corresponds to $t = 2$ is $(2,0)$. (Note: Each part of the curve that looks like a line segment is a line segment.)**



The first line segment is $y = x$. The second line segment is $y = 2 - x$. We may as well let $x = t$. The parameterization is

$$\boxed{x(t) = t \text{ for } 0 \le t \le 2 \quad \text{and} \quad y(t) = \begin{cases} t & \text{for } 0 \le t \le 1 \\ 2 - t & \text{for } 1 < t \le 2 \end{cases}}$$

2. (4 points) **Express** $\sin(x+h)$ **in terms of** $\sin x$, $\sin h$, $\cos x$, **and** $\cos h$.

One of the five trig facts is:

$$\boxed{\sin(x+h) = \sin x \cos h + \cos x \sin h.}$$

3. (5 points) **Let** $f(x) = \sqrt{x}$. **Find** $\lim\limits_{a \to b} \frac{f(a) - f(b)}{a - b}$.

We see that

$$\lim_{a\to b}\frac{f(a)-f(b)}{a-b}=\lim_{a\to b}\frac{\sqrt{a}-\sqrt{b}}{a-b}.$$

Multiply top and bottom by the conjugate of $\sqrt{a}-\sqrt{b}$ to obtain

$$\lim_{a\to b}\frac{f(a)-f(b)}{a-b}=\lim_{a\to b}\frac{a-b}{(a-b)(\sqrt{a}+\sqrt{b})}=\lim_{a\to b}\frac{1}{(\sqrt{a}+\sqrt{b})}=\frac{1}{\sqrt{b}+\sqrt{b}}=\boxed{\frac{1}{2\sqrt{b}}.}$$

4. (5 points) **Let** $f(x)=\sqrt{3x+1}$ **. Use the DEFINITION OF THE DERIVATIVE to find** $f'(x)$ **.**

We see that

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h}.$$

Multiply top and bottom by the conjugate of $\sqrt{3(x+h)+1}-\sqrt{3x+1}$ to obtain

$$f'(x)=\lim_{h\to 0}\frac{3(x+h)+1-(3x+1)}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}=\lim_{h\to 0}\frac{3h}{h(\sqrt{3(x+h)+1}+\sqrt{3x+1})}.$$

Divide top and bottom by h to obtain

$$f'(x)=\lim_{h\to 0}\frac{3}{\sqrt{3(x+h)+1}+\sqrt{3x+1}}=\frac{3}{\sqrt{3x+1}+\sqrt{3x+1}}=\boxed{\frac{3}{2\sqrt{3x+1}}.}$$

5. (4 points) **Compute** $\lim_{h\to 0}\frac{\cos h-1}{h}$ **.**

Multiply top and bottom by $\cos h+1$ to see that

$$\lim_{h\to 0}\frac{\cos h-1}{h}=\lim_{h\to 0}\frac{\cos^2 h-1}{h(\cos h+1)}=\lim_{h\to 0}\frac{-\sin^2 h}{h(\cos h+1)}=\lim_{h\to 0}\frac{\sin h}{h}\frac{(-\sin h)}{\cos h+1}.$$

In class we saw that the limit of the first factor is 1. The limit of the second factor is $\frac{0}{1+1}=0$. We conclude that

$$\lim_{h\to 0}\frac{\cos h-1}{h}=\boxed{0.}$$

6. (4 points) **Compute** $\lim_{x\to\infty}\left(x+\frac{2}{x}\right)^{3x}$ **.**

There is nothing subtle about this problem. As x goes to infinity, the base goes to infinity and the exponent goes to infinity, so the whole expression $\left(x+\frac{2}{x}\right)^{3x}$ goes to $\boxed{+\infty}$.

7. (4 points) **Find the equation of the line tangent to $f(x) = x^{10} + x$ at $x = 1$.**

We know $f'(x) = 10x^9 + 1$; so $f'(1) = 11$. We also know $f(1) = 2$. The line passing through $(1, 2)$ with slope 11 is $\boxed{y - 2 = 11(x - 1).}$

8. (5 points) **The height of an object above the ground is given by $y(t) = -16t^2 + 32t + 48$, where y is measured in feet and t is measured in seconds. Find the velocity of the object when it hits the ground. Be sure to give units.**

The object hits the ground when $y(t) = 0$; so, $-16(t^2 - 2t - 3) = 0$; so, $-16(t - 3)(t + 1) = 0$. The object hits the ground when $t = -1$ or $t = 3$. Of course, $t = -1$ is not relevant. The object hits the ground when $t = 3$. The velocity of the object is given by $v(t) = y'(t) = -32t + 32$. The velocity of the object when it hits the ground is $v(3) = -32(3) + 32 = \boxed{-64 \text{ ft/sec}}$

9. (4 points) **Let $f(x) = \frac{2}{x} + 3x^2 + \sqrt{2x}$. Find $f'(x)$.**

Write $f(x)$ as $f(x) = 2x^{-1} + 3x^2 + \sqrt{2}x^{1/2}$. Now take the derivative to obtain

$$\boxed{f'(x) = -2x^{-2} + 6x + (1/2)\sqrt{2}x^{-1/2}.}$$

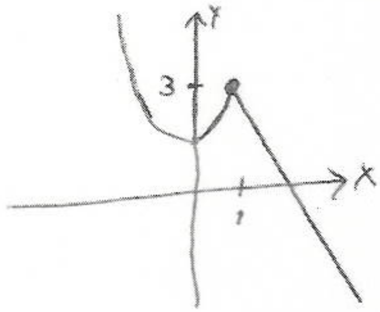
10. (5 points) **Let $f(x) = \frac{2x^2+4x}{\sqrt{x}+2x}$. Find $f'(x)$.**

Again, keep in mind that $\sqrt{x} = x^{1/2}$. Apply the quotient rule to see that

$$\boxed{f'(x) = \frac{(\sqrt{x} + 2x)(4x + 4) - (2x^2 + 4x)((1/2)x^{-1/2} + 2)}{(\sqrt{x} + 2x)^2}.}$$

11. (5 points) **Consider the function** $f(x) = \begin{cases} x^2+2 & \textbf{if } x \le 1 \\ 4-x & \textbf{if } 1 < x. \end{cases}$

(a) **Graph** $y = f(x)$ **.**



(b) **Is** $f(x)$ **continuous at** $x = 1$ **? Explain.**

Yes. The graph can be drawn without lifting one's pencil. That is,

$$\lim_{x\to 1^-} f(x) = 3, \quad \lim_{x\to 1^+} f(x) = 3, \quad \text{and} \quad f(1) = 3.$$

(c) **Is** $f(x)$ **differentiable at** $x = 1$ **? Explain.**

No. There is a sharp turn at $x = 1$. In other words, $f'(1)$ does not exist since

$$\lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = 2 \quad \text{but} \quad \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = -1.$$