Exam 2, Math 141, 1996

PRINT Your Name:_____ Section:_____

There are 10 problems on 5 pages. Each problems is worth 10 point. In problem 2 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer.
NO CALCULATORS!

1. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 2 - x & \text{if } x < 0, \\ 2 + x & \text{if } 0 \le x \le 1, \text{ and} \\ 3 - x^2 & \text{if } 1 < x. \end{cases}$$

   (a) Graph $y = f(x)$.
   (b) Fill in the blanks:

$$f(0) = \_\_ \quad \lim_{x\to 0^+} f(x) = \_\_ \quad \lim_{x\to 0^-} f(x) = \_\_ \quad \lim_{x\to 0} f(x) = \_\_$$
$$f(1) = \_\_ \quad \lim_{x\to 1^+} f(x) = \_\_ \quad \lim_{x\to 1^-} f(x) = \_\_ \quad \lim_{x\to 1} f(x) = \_\_$$
$$f(2) = \_\_ \quad \lim_{x\to 2^+} f(x) = \_\_ \quad \lim_{x\to 2^-} f(x) = \_\_ \quad \lim_{x\to 2} f(x) = \_\_$$

   (c) Where is $f(x)$ continuous?
   (d) Where is $f(x)$ differentiable?

2. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = 4\sqrt{2x-3}$.

3. Find the equation of the line tangent to $f(x) = x^5 - 3x^2$ at $x = 2$.

4. The position of an object above the surface of the earth is given by $s(t) = -16t^2 + 64t + 100$, where s is measured in feet and t is measured in seconds. How high does the object get?

5. Let $y = x^2 \cos^2(4x^5 + 19x)$. Find dy.

6. Let $y = \sqrt{4x^3 + 9x + \sin^3(\cos(5x^4 + 3x))}$. Find $\frac{dy}{dx}$.

7. Let $3x^2y^3 = \sin(xy^2) + 3x^5$. Find $\frac{dy}{dx}$.

8. Let $y = \frac{3}{x} + 15 - 4\sqrt{x}$. Find $\frac{d^2y}{dx^2}$.

9. The area of a square is growing at the rate of 4 square feet per second. How fast is the length of each side growing when each side has length 6 feet?

10. A 30 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 3 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 5 feet from the wall?