Math 141, 1995, Exam 4

PRINT Your Name:_____

There are 13 problems on 7 pages. Problems 1 and 2 are each worth 6 points. Each of the other problems is worth 8 points. SHOW your work. $\boxed{CIRCLE}$ your answer. You might find the following formulas to be useful:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

NO CALCULATORS!

1. State both parts of the Fundamental Theorem of Calculus.

2. Define the definite integral.

3. Let $y = \sqrt{x\cos^3(4x^2+3) + \sin^4(x)}$. Find $\frac{dy}{dx}$.

4. Find $\int \frac{2}{x^2} + \sin(2x)dx$.

5. Find $\int \frac{\sin x \cos x}{\sqrt{2\sin^2 x + 1}} dx$.

6. Let $f(x) = \frac{x^2 - 2x + 4}{x - 2}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. Graph $y = f(x)$.

7. The surface area of a cube is growing at the constant rate of 1000 square inches per second. How fast is the volume growing when each edge is 5 inches long?

8. Find the points on the curve $y^2 + 2x = 9$ which are closest to the point $(0, 0)$.

9. Solve the Initial Value Problem $\frac{dy}{dx} = x^3y^2$, $y(2) = 1$.

10. Let $f(x) = x^2 + x$. Simplify the expression $\sum_{i=1}^{n} f(\frac{3i}{n})$. Your answer is not allowed to have a summation sign or $\ldots$.

11. Find the **exact** amount of area inside the following 50 **boxes**. The base of each box has the same size.

12. Find the area of region between $x - 2y = 0$ and $y^2 - 2x = 0$.

13. Find the volume of the solid which is obtained by revolving the region of problem 12 about the $x-$ axis.