Math 141, 1995, Exam 2

PRINT Your Name:_____

There are 13 problems on 6 pages. In problem 10 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer.

1. (10 points – The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} 4 - x & \text{if } x < 2, \\ 3 & \text{if } x = 2, \\ x - 1 & \text{if } 2 < x < 3, \text{ and} \\ x^2 - 8 & \text{if } 3 \leq x. \end{cases}$$

(a) Graph $y = f(x)$.

(b) Fill in the blanks:

$f(1) = \underline{}$ $\quad \lim\limits_{x \to 1^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 1^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 1} f(x) = \underline{}$

$f(2) = \underline{}$ $\quad \lim\limits_{x \to 2^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 2^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 2} f(x) = \underline{}$

$f(3) = \underline{}$ $\quad \lim\limits_{x \to 3^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 3^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 3} f(x) = \underline{}$

2. (7 points) Let $y = \frac{1}{\sqrt{2x}} - \sin(2x)$. Find $\frac{dy}{dx}$.

3. (10 points – The penalty for each mistake is four points.) The picture represents the graph of $y = f(x)$. Fill in the blanks:

$f(1) = \underline{}$ $\quad \lim\limits_{x \to 1^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 1^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 1} f(x) = \underline{}$

$f(2) = \underline{}$ $\quad \lim\limits_{x \to 2^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 2^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 2} f(x) = \underline{}$

$f(3) = \underline{}$ $\quad \lim\limits_{x \to 3^+} f(x) = \underline{}$ $\quad \lim\limits_{x \to 3^-} f(x) = \underline{}$ $\quad \lim\limits_{x \to 3} f(x) = \underline{}$

4. (7 points) Let $y = (2x^3 + \sqrt{2}x)^4(2x^5 + \cos(3x))^6$. Find $\frac{dy}{dx}$.

5. (7 points) Let $y = \dfrac{4x^5 + \frac{2}{x} + 19}{8x^3 + 15x + 6}$. Find $\frac{dy}{dx}$.

6. (7 points) Let $4xy^2 + \sin(xy) = 3y^2 + 6x^2$. Find $\frac{dy}{dx}$.

7. (7 points) Let $y = \sqrt{\sin^2(4x^2 + 3x + 19) + \cos^3(x)}$. Find $\frac{dy}{dx}$.

8. (8 points) A cube is growing at the constant rate of 1000 cubic inches per second. How fast is each edge growing when each edge is 5 inches long?

9. (7 points) Find the equation of the line tangent to $y = 3x^5 + 4x + 2$ when $x = 1$.

10. (7 points) Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \sqrt{2x + 1}$.

11. (8 points) Find the equation of every line which passes through $(-1, -1)$ and is also tangent $y = x^2 + 2x + 4$.

12. (7 points) The position of an object above the earth's surface is given by

$$s(t) = -16t^2 + 48t + 64.$$

What is the velocity of the object when it strikes the ground?

13. (8 points) A student is using a straw to drink from a conical cup, whose axis is vertical, at the rate of 3 cubic inches per second. If the height of the cup is 12 inches and the radius of its opening is 8 inches, how fast is the level of the liquid falling when the depth of the liquid is 7 inches? (Recall that the volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)