

**Math 141, 1995, Exam 1**

PRINT Your Name: \_\_\_\_\_

There are 13 problems on 4 pages. Problem 2 is worth 12 points; problem 3 is worth 11 points; each of the other problems is worth 7 points. In problem 7 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. *CIRCLE* your answer.

1. Let  $f(x) = 2x + 1$  and  $g(x) = 3x^2$ .

(a) Find  $(f \circ g)(x)$ .

(b) Find  $(g \circ f)(x)$ .

2. (The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} x & \text{if } x < 1, \\ 3 - x & \text{if } 1 \leq x \leq 2, \text{ and} \\ x^2 - 3 & \text{if } 2 < x. \end{cases}$$

(a) Graph  $y = f(x)$ .

(b) Fill in the blanks:

$$\begin{array}{cccc} f(1) = \underline{\quad} & \lim_{x \rightarrow 1^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 1^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 1} f(x) = \underline{\quad} \\ f(2) = \underline{\quad} & \lim_{x \rightarrow 2^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 2^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 2} f(x) = \underline{\quad} \\ f(3) = \underline{\quad} & \lim_{x \rightarrow 3^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 3^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 3} f(x) = \underline{\quad} \end{array}$$

3. (The penalty for each mistake is four points.) The picture represents the graph of  $y = f(x)$ .

(a) Fill in the blanks:

$$\begin{array}{cccc} f(1) = \underline{\quad} & \lim_{x \rightarrow 1^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 1^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 1} f(x) = \underline{\quad} \\ f(2) = \underline{\quad} & \lim_{x \rightarrow 2^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 2^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 2} f(x) = \underline{\quad} \\ f(3) = \underline{\quad} & \lim_{x \rightarrow 3^+} f(x) = \underline{\quad} & \lim_{x \rightarrow 3^-} f(x) = \underline{\quad} & \lim_{x \rightarrow 3} f(x) = \underline{\quad} \end{array}$$

(b) Where is  $f$  discontinuous?

(c) Where is  $f$  not differentiable?

4. Graph  $y = (x - 3)^2 + 1$ .

5. Find the angle (in radians) between  $y = x$  and  $y = 2x$ .

6. Express  $\sin(x - y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$ , and  $\cos y$ .

7. Use the DEFINITION of the DERIVATIVE to find the derivative of  $f(x) = \sqrt{x}$ .
8. Find the equation of the line tangent to  $f(x) = 4x^4 + 3x^2$  at the point  $(1, 7)$ .
9. Find  $\lim_{x \rightarrow 0} \frac{\sin x}{x^3}$ .
10. Find  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x - 1}$ .
11. Find  $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{(x + 1)^2}$ .
12. Find  $\lim_{x \rightarrow 1} \frac{x^2 + 3x + 2}{(x - 1)^2}$ .
13. Let  $f(x) = (2x^3 + 3x)(8x^2 + 19x + 1)$ . Find  $f'(x)$ .