Math 141, Final Exam, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 23 problems. Problems 1 through 7 are worth 8 points each. Each of the other problems is worth 9 points. The exam is worth 200 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**. Otherwise, get your grade from VIP.

You might find the following formulas to be useful:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

I will post the solutions on my website a few hours after the exam is finished.

1. Let $y = 2^x$. Find $\frac{dy}{dx}$.

2. Let $y = \cos(\cos x)$. Find $\frac{dy}{dx}$.

3. Let $y = x^2(\arcsin x)^3$. Find $\frac{dy}{dx}$.

4. Let $y = \sin x \left(\int_0^x \sin(t^2)dt \right)$. Find $\frac{dy}{dx}$.

5. Find $\int_1^2 x^2 dx$.

6. Find $\int_{\frac{\pi}{12}}^{\frac{\pi}{9}} \sec^2 3\theta d\theta$.

7. Find $\int_{-1}^{1} \frac{x^2 dx}{\sqrt{x^3+9}}$.

8. Find $\int_0^{\frac{\sqrt{\pi}}{2}} 5x \cos(x^2) dx$.

9. Find $\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2$.

10. Find $\lim_{x\to\infty} x^2 - \sqrt{x^4 + 6x^2}$.

11. Find $\lim_{x\to 0^+} x^{\frac{5}{1+\ln x}}$.

12. Use the DEFINITION of the derivative to find $f'(x)$ for $f(x) = \frac{1}{\sqrt{2x-3}}$.

13. Parameterize the diamond with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.

14. The position of an object at time t is given by

$$\begin{cases} x = 4\sin t \\ y = 3\cos t. \end{cases}$$

(a) Eliminate the parameter to find a Cartesian equation for the path of the object.
(b) Graph the path of the object.
(c) On your graph, mark the position of the object at a few particular values for time.

15. Let $f(x) = 4x^{1/3} - x^{4/3}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. Graph $y = f(x)$.

16. Each edge of a cube is increasing at the rate of 4 inches per second. How fast is the surface area of the cube increasing when an edge is 12 inches long?

17. Consider the right circular cylinder of greatest volume that can be inscribed in a right circular cone. What is the ratio of the volume of the cylinder divided by the volume of the cone ?

18. State the Mean Value Theorem.

19. Consider the region bounded by $y = x^2$, $x = 1$, $x = 2$, and the x-axis. Partition the base into 50 subintervals of equal size. Over each subinterval, imagine a rectangle which approximates, but OVER estimates, the area under the curve. How much area is inside your 50 rectangles? (You must answer the question I asked, not some other question. I expect an exact answer in closed form: no dots and no summation signs.)

20. The position of an object above the surface of the earth is given by

$$s(t) = -16t^2 + 64t + 100,$$

where s is measured in feet and t is measured in seconds. How high does the object get?

21. State BOTH parts of the Fundamental Theorem of Calculus.

22. Let a and b be real numbers with $\frac{-\pi}{2} < a < b < \frac{\pi}{2}$. Prove that $\tan b - \tan a \geq b - a$.

23. Find the equations of the lines through the origin that are tangent to $2x^2 - 4x + y^2 + 1 = 0$.