Math 141, Exam 4, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 12 problems. Problems 1 through 4 are worth 9 points each. Problems 5 through 12 are worth 8 points each. The exam is worth 100 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website a few hours after the exam is finished.

1. Find $\int \frac{dx}{e^x}$. **Check your answer.**

2. Find $\int \sec 4x \, \tan 4x \, dx$. **Check your answer.**

3. Find $\int \frac{\sec^2 x dx}{\sqrt{1-\tan^2 x}}$. **Check your answer.**

4. Find $\lim_{\Delta x \to 0} \frac{\ln(e^2+\Delta x)-2}{\Delta x}$.

5. Find $\lim_{x \to 0^+} x^{\frac{\ln 2}{1+\ln x}}$.

6. Find $\lim_{x \to +\infty} \frac{x^3}{e^{-x}}$.

7. Find $\frac{dy}{dx}$ for $\sin(x^2y^2) = x$.

8. Find $\frac{dy}{dx}$ for $y = \ln(\sin^2 x)$.

9. Find $\frac{dy}{dx}$ for $y = x^{\sin x}$.

10. Find the coordinates of the point P on the curve

$$y = \frac{1}{x^2} \quad \text{for } x > 0$$

where the segment of the tangent line at P that is cut off by the coordinate axes has its shortest length.

11. Let $f(x) = x^2 \ln x$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local maximum points, local minimum points, and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. What is the domain of $f(x)$? Graph $y = f(x)$.

12. A boat is pulled into a dock by means of a rope attached to a pulley on the dock. (See the picture.) The rope is attached to the bow of the boat at a point 10 feet below the pulley. If the rope is pulled through the pulley at a rate of 20 feet/minute, at what rate will the boat be approaching the dock when 125 feet of rope are out?