Math 141,   Exam 3,   Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 14 problems. Problem 1 is worth 9 points. Each other problem is worth 7 points. The exam is worth 100 points. SHOW your work. Make your work be coherent and clear. Write in complete sentences whenever this is possible. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website a few hours after the exam is finished.

1. Let $f(x) = \begin{cases} x^2 & \text{for } x < 1, \\ 2x - 1 & \text{for } 1 \le x \le 2 \\ 5 - x & \text{for } 2 < x < 3 \\ x & \text{for } 3 \le x. \end{cases}$

   (a) Graph $y = f(x)$.
   (b) Find

$$\lim_{x \to 0^+} f(x) \qquad \lim_{x \to 0^-} f(x) \qquad \lim_{x \to 0} f(x) \qquad f(0)$$
$$\lim_{x \to 1^+} f(x) \qquad \lim_{x \to 1^-} f(x) \qquad \lim_{x \to 1} f(x) \qquad f(1)$$
$$\lim_{x \to 2^+} f(x) \qquad \lim_{x \to 2^-} f(x) \qquad \lim_{x \to 2} f(x) \qquad f(2)$$
$$\lim_{x \to 3^+} f(x) \qquad \lim_{x \to 3^-} f(x) \qquad \lim_{x \to 3} f(x) \qquad f(3)$$
$$\lim_{x \to 4^+} f(x) \qquad \lim_{x \to 4^-} f(x) \qquad \lim_{x \to 4} f(x) \qquad f(4)$$

   (c) Where is $f(x)$ continuous?
   (d) Where is $f(x)$ differentiable?

2. Find the derivative of $f(x) = 3x^2 + \sqrt{2x} + \frac{1}{4x} + e$.

3. Parameterize the triangle whose vertices are $(1,0)$, $(0,1)$, and $(-1,0)$.

4. Use **the definition of the derivative** to find the derivative of $f(x) = \sqrt{2x - 3}$.

5. Find $\lim\limits_{n \to \infty} \left(\frac{n}{n-3} \right)^n$.

6. Find $\lim\limits_{x \to 0} \frac{1 - \cos 3x}{x^2}$.

7. Find $\lim\limits_{x \to \infty} \sqrt{x^6 + 5x^3} - x^3$.

8. Find $\lim\limits_{x \to 1} \frac{x^6 - 1}{x - 1}$.

9. Find the equation of the line tangent to $x^4 + y^4 = 16$ at $(1, \sqrt[4]{15})$.

10. Find the derivative of $f(x) = \sin(\ln(2x^2 + 3x))$.

11. Find the derivative of $f(x) = e^{3x^2 + 2x} \tan x$.

12. Find the x-coordinates of all points on the graph of $y = 1 - x^2$ at which the tangent line passes through the point $(2, 0)$.

13. The height of an object above the ground at time t is $s(t) = -16t^2 + 32t + 48$, where s is measured in feet and t is measured in seconds. What is the velocity of the object when it strikes the ground?

14. A cube is growing at the constant rate of 1000 cubic inches per second. How fast is the surface area growing when each edge is 5 inches long?