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PRINT Your Name:______________________________ Section:__________

There are 8 problems on 4 pages. Problem 4 is worth 30 points. Each of the other problems is worth 10 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. NO **CALCULATORS!**

1. State both parts of the Fundamental Theorem of Calculus.

Let f(x) be a continuous function.

a) If $A(x) = \int_a^x f(t)\,dt$, then $A'(x) = f(x)$.

b) If $F'(x) = f(x)$, then $\int_a^b f(x)\,dx = F(b) - F(a)$.

2. Let $A(x) = \int\limits_1^x \frac{1}{t}\,dt$. Find $A'(x)$.

$A'(x) = \frac{1}{x}$

3. Compute ~~$A(x) =$~~ $\int\limits_0^{\pi/2} \sin^4 x \cos x \, dx$.

$$+ \frac{\sin^5 x}{5} \Big]_0^{\frac{\pi}{2}} = \frac{1}{5}$$

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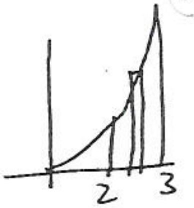
4. (10 points for each part) Consider the region R which is bounded by $y = x^2$, $x = 2$, $x = 3$, and $y = 0$.

(a) Find the ~~volume~~ area of R.

$$\int_2^3 x^2\,dx = \frac{x^3}{3}\Big]_2^3 = \boxed{9 - \frac{8}{3}}$$

(b) Find the volume of the solid which is obtained by revolving R about the $x-$ axis.



discs $\pi r^2 t$ $t = dx$ $r = x^2$

$$Vol = \pi \int_2^3 x^4\,dx = \pi \frac{x^5}{5}\Big]_2^3 = \boxed{\frac{\pi}{5}\left(3^5 - 32\right)}$$

(c) Find the volume of the solid which is obtained by revolving R about the $y-$ axis.

shells $2\pi r h t$

$t = dx$
$r = x$
$h = x^2$

$$Vol = 2\pi \int_2^3 x^3\,dx = 2\pi \frac{x^4}{4}\Big]_2^3$$

$$= \boxed{\frac{\pi}{2}\left(3^4 - 16\right)}$$

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5. Let $f(x) = -x^3 + 3x^2$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Graph $y = f(x)$.

$f' = -3x^2 + 6x$

$= -3x(x-2)$

$f'' = -6x + 6$

$= -6(x-1)$

f'neg | f'pos | f'Neg

0 2

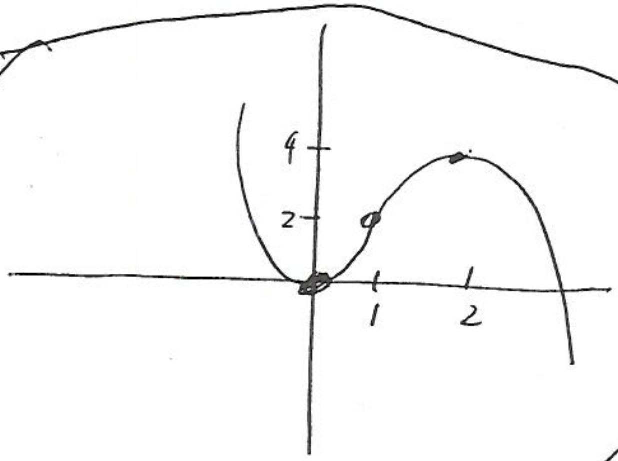
f'pos | f'Neg

1

$f(0) = 0$

$f(1) = 2$

$f(2) = -8 + 12 = 4$



inc for $0 < x < 2$

dec for $x < 0$ also $2 < x$

cu for $x < 1$

cd for $1 < x$

loc. max (2, 4)

loc min (0, 0)

p of i (1, 2)

6. Find the length of $y = 2x^{3/2}$ between $x = 1/3$ and $x = 7$.

$$\text{length} = \int_{\frac{1}{3}}^{7} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_{\frac{1}{3}}^{7} \sqrt{1+(3\sqrt{x})^2}\,dx = \int_{\frac{1}{3}}^{7} \sqrt{1+9x}\,dx$$

$$= \frac{1}{9}\left(\frac{2}{3}\right)(1+9x)^{\frac{3}{2}}\Big]_{\frac{1}{3}}^{7} = \frac{2}{27}\left(64^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = \frac{2}{27}\left(8^3 - 2^3\right)$$

$$= \frac{2 \cdot 8}{27}(64-1) = \boxed{\frac{16}{27}(63)}$$

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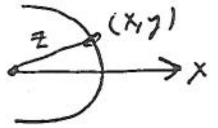
7. Find the points on the curve $y^2 + 2x = 9$ which are closest to the point $(0, 0)$.



Minimize $z = x^2 + y^2$ on $y^2 = 9 - 2x$

So minize $z = x^2 + 9 - 2x$

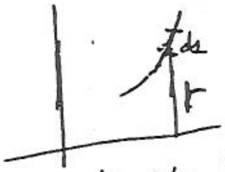
$$\frac{dz}{dx} = 2x - 2$$

$\frac{dz}{dx} = 0$ when $x = 1$

$$\boxed{\text{ans } (1, \sqrt{7}) \text{ and } (1, -\sqrt{7})}$$

8. Find the area of the surface which is generated by revolving $y = x^3/3$, for $1 \le x \le \sqrt{7}$, about the $x-$ axis.

$$\text{area} = \int_1^{\sqrt{7}} 2\pi r\, ds = \int_1^{\sqrt{7}} \frac{2\pi}{3} x^3 \sqrt{1+x^4}\, dx = \frac{\cancel{2}\pi}{3} \frac{\cancel{2}}{3} \frac{(1+x^4)^{\frac{3}{2}}}{\cancel{4}} \Bigg]_1^{\sqrt{7}}$$



$$r = y = x^3/3$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \sqrt{1 + (x^2)^2}$$

$$= \boxed{\frac{\pi}{9}\left(50^{\frac{3}{2}} - 2^{\frac{3}{2}}\right)}$$