1999 EX3 73

PRINT Your Name:____________________ Recitation Time ____________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!**

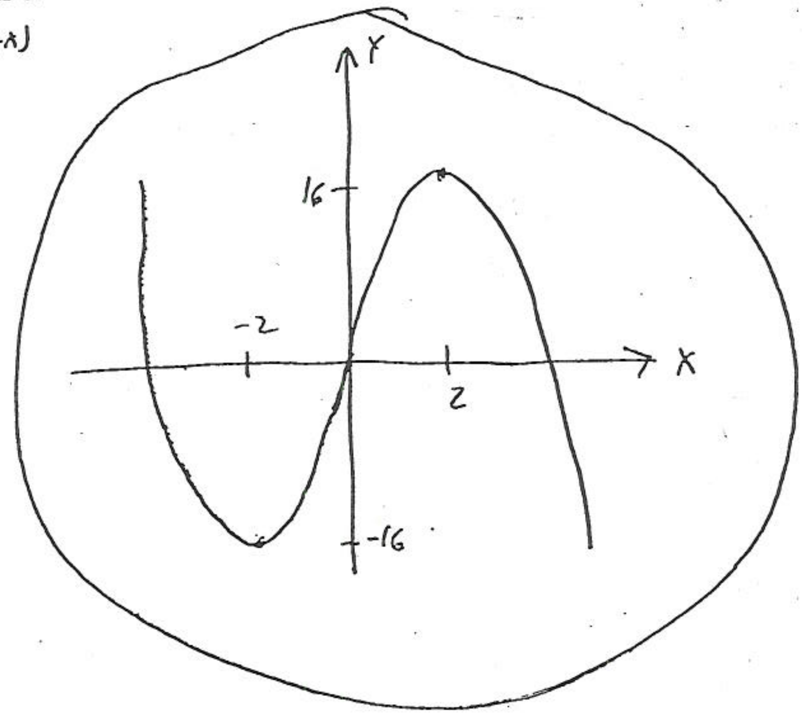
1. Let $f(x) = 12x - x^3$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. Graph $y = f(x)$.

$f' = 12 - 3x^2 = 3(4 - x^2) = 3(2-x)(2+x)$

$f'' = -6x$

f' neg, f' pos, f' neg; -2, 2

f'' pos, f'' neg; 0



f is inc for $-2<x<2$

f is dec for $x<-2$ also for $2<x$

f is c.u for $x<0$

f is c.d for $0<x$

$(0,0)$ is p.o.i

$(2,16)$ is local max

$(-2,-16)$ is local min

$\lim_{x\to\infty} f = -\infty$ No vert asy

$\lim_{x\to-\infty} f = +\infty$ No horizontal asy

2. State the Mean Value Theorem. If $f(x)$ is differentiable for all x with $a \le x \le b$, then there exists a number c with $a \le c \le b$ ~~with~~ and $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.

3. Find $\int \frac{1}{x^2} + \sin 2x\, dx$ $= -\dfrac{1}{x} - \dfrac{\cos 2x}{2} + C$

1999 Ex3 74

4. Let $f(x) = \dfrac{1}{1+x^2}$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. Graph $y = f(x)$.

No V.A. because denom never 0

$\lim_{x \to \pm\infty} f = 0$ so $y=0$ is HA

$f' = \dfrac{-2x}{(1+x^2)^2} = -2x(1+x^2)^{-2}$

$f'' = -2x(-2)(1+x^2)^{-3}(2x) - 2(1+x^2)^{-2}$

$= 2(1+x^2)^{-3}\left(4x^2 - (1+x^2)\right)$

$= \dfrac{2(3x^2-1)}{(1+x^2)^3}$

f' pos | 0 | f' neg

f'' pos | $-\frac{1}{\sqrt{3}}$ | f'' neg | $\frac{1}{\sqrt{3}}$ | f'' pos

f is dec for $0<x$

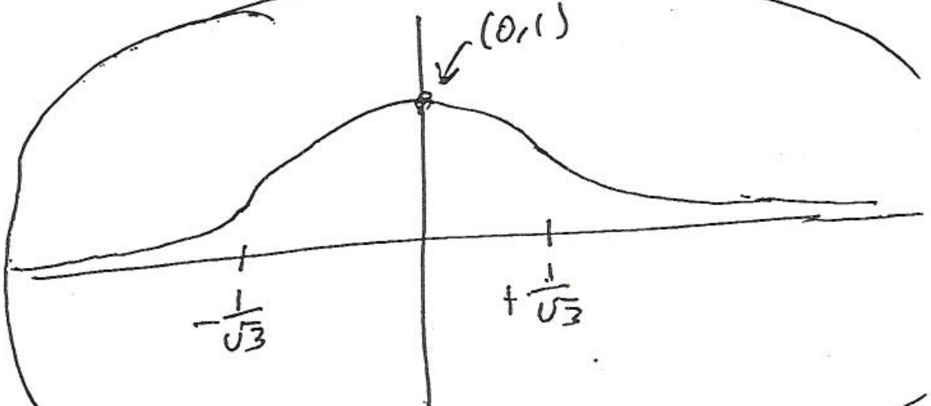
f is inc. for $x<0$

f is c.u. for $\frac{1}{\sqrt{3}}<x$ also for $x<-\frac{1}{\sqrt{3}}$

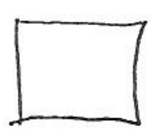
f is c.d. for $-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

$\left(\frac{1}{\sqrt{3}}, f\left(\frac{1}{\sqrt{3}}\right)\right)$ and $\left(-\frac{1}{\sqrt{3}}, f\left(-\frac{1}{\sqrt{3}}\right)\right)$ are p.

$(0,1)$ is a loc. max



5. The area of a square is growing at the rate of 10 square inches per second. How fast is each side growing when each side has length 3 inches?

s 

$A = s^2$

$\dfrac{dA}{dt} = 10$

Find $\left.\dfrac{ds}{dt}\right|_{s=3}$

$\dfrac{dA}{dt} = 2s\dfrac{ds}{dt}$

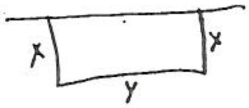
$\dfrac{ds}{dt} = \dfrac{1}{2s}\dfrac{dA}{dt}$

$\left.\dfrac{ds}{dt}\right|_{s=3} = \dfrac{10}{6} = \dfrac{5}{3}$ in/sec

1999 Ex3

75

6. Farmer Brown has 80 feet of fence with which he plans to enclose a rectangular pen along one side of his 100-foot barn. (The side along the barn needs no fence.) What are the dimensions of the pen that has maximum area?



$2x + y = 80$

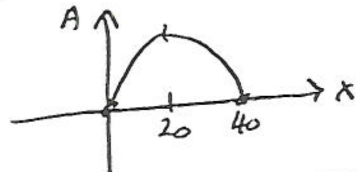
Maximize $A = xy$

$A = x(80 - 2x)$

$A = 80x - 2x^2 \quad 0 \le x \le 40$

$A' = 80 - 4x$

$A' = 0$ when $x = 20$



The pen has maximum area when $x = 20$ and $y = 40$.

7. Let $2x^3y^2 = \sin(2x^2y^4)$. Find $\frac{dy}{dx}$.

$2x^3 2y \frac{dy}{dx} + 6x^2y^2 = \cos(2x^2y^4)\left[2x^2 4y^3 \frac{dy}{dx} + 4xy^4\right]$

$\left[4x^3y - 8x^2y^3\cos(2x^2y^4)\right]\frac{dy}{dx} = 4xy^4\cos(2x^2y^4) - 6x^2y^2$

$$\frac{dy}{dx} = \frac{4xy^4\cos(2x^2y^4) - 6x^2y^2}{4x^3y - 8x^2y^3\cos(2x^2y^4)}$$

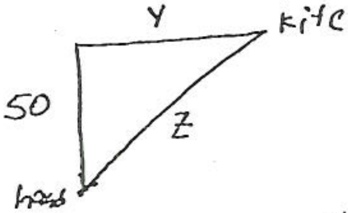
$6x + 4 = 9x + 15$

Find x

1999 Ex3

76

8. A child is flying a kite. If the kite is 50 feet above the the child's hand level and the wind is blowing it on a horizontal course at 5 feet per second, how fast is the child paying out cord when 100 feet of cord is out. (Assume that the cord forms a line.)



We know $\frac{dy}{dt} = 5$ ft/s

We want $\left.\frac{dz}{dt}\right|_{z=100}$

$50^2 + y^2 = z^2$

$2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{y}{z}\frac{dy}{dt}$

when $z = 100$ $y = \sqrt{100^2 - 50^2}$

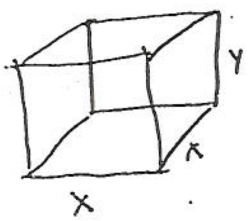
$\left.\frac{dz}{dt}\right|_{z=100} = \frac{\sqrt{100^2-50^2}}{100}\,5$ ft/s $= \frac{\sqrt{7500}}{20} = \frac{50\sqrt{3}}{20} = \frac{5\sqrt{3}}{2}$

9. Let $y = \sqrt{x^3\cos^2(2x) + 19x^2}$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{38x + 2x^3\cos(2x)(-\sin(2x))2 + 3x^2\cos^2(2x)}{2\sqrt{x^3\cos^2(2x)+19x^2}}$$

77

10. A rectangular box with a square base is to be constructed to hold 12,000 cubic feet of water. If the metal top costs twice as much per square foot as the concrete sides and base, what are the most economical dimensions for the box?



$x^2 y = 12{,}000$

Let each sq ft of concrete cost $d

So each sq ft of metal costs $2d

$$C = \underbrace{2dx^2}_{\text{bot}} + \underbrace{4dxy}_{\text{sides}} + \underbrace{2dx^2}_{\text{top}}$$

$$C(x) = d\left(3x^2 + 4x\frac{12{,}000}{x^2}\right)$$

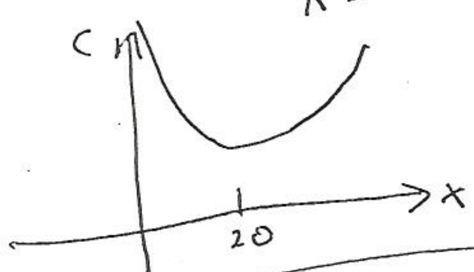
$$C(x) = d\left(3x^2 + \frac{48000}{x}\right), \quad 0 < x$$

$$C' = d\left(6x - \frac{48000}{x^2}\right)$$

$C' = 0$ when $6x^3 = 48000$

$x^3 = 8000$

$x = 20$



To minimize cost take $x = 20$ ft and $y = \frac{12000}{400} = 30$ ft