Exam 3 141 2000

PRINT Your Name:____________________ Recitation Time _________ Tu. Th.
There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Let $3x^2y^4 = \cos(4x^2y^5)$. Find $\frac{dy}{dx}$.

$$3x^2 4y^3 \frac{dy}{dx} + 6xy^4 = -\sin(4x^2y^5)\left[4x^2 5y^4 \frac{dy}{dx} + 8xy^5\right]$$

$$\frac{dy}{dx}\left[12x^2y^3 + \sin(4x^2y^5)\, 20x^2y^4\right] = -6xy^4 - 8xy^5\sin(4x^2y^5)$$

$$\frac{dy}{dx} = \frac{-6xy^4 - 8xy^5\sin(4x^2y^5)}{12x^2y^3 + 20x^2y^4\sin(4x^2y^5)}$$

2. Let $y = 3x^2\sin^2(x^3)$. Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 3x^2\, 2\sin(x^3)\cos(x^3)\, 3x^2 + 6x\sin^2(x^3)$$

3. Let $f(x) = 3x - x^3$. Find all vertical and horizontal asymptotes of $y = f(x)$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find all local maximum points, local minimum points, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

No horizontal or Vertical asymptotes

$f' = 3 - 3x^2 = 3(1-x)(1+x)$

$f'' = -6x$

$- - - \;|_{-1}\; + + + \;|_{1}\; - - -$ $\quad f'$

$+ + + \;|_{0}\; - - -$ $\quad f''$

f is increasing for $-1 < x < 1$

f is decreasing for $x < -1$ and also for $1 < x$

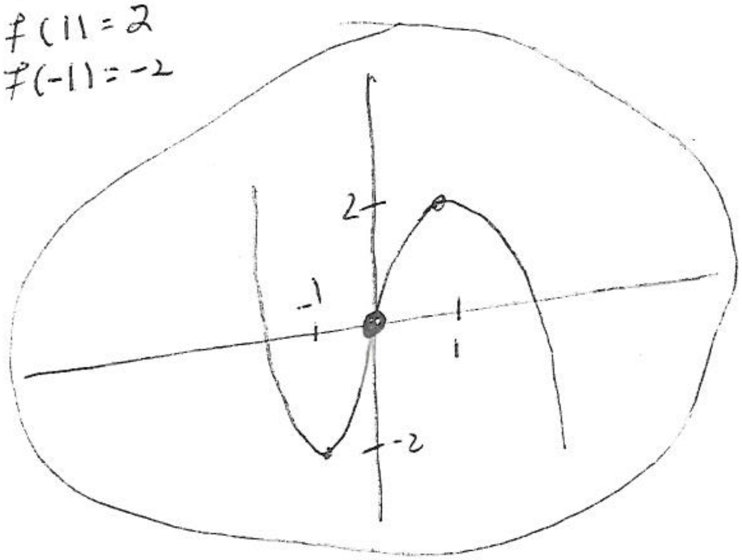
f is concave up for $x < 0$

f is concave down for $0 < x$

$f(0) = 0$

$f(1) = 2$

$f(-1) = -2$



4. Find $\int x(x+1)dx$. $= \int (x^2 + x)dx = \frac{x^3}{3} + \frac{x^2}{2} + c$

5. Let $f(x) = \frac{x}{x^2-4}$. Find all vertical and horizontal asymptotes of $y = f(x)$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? Find all local maximum points, local minimum points, and points of inflection of $y = f(x)$. Graph $y = f(x)$.

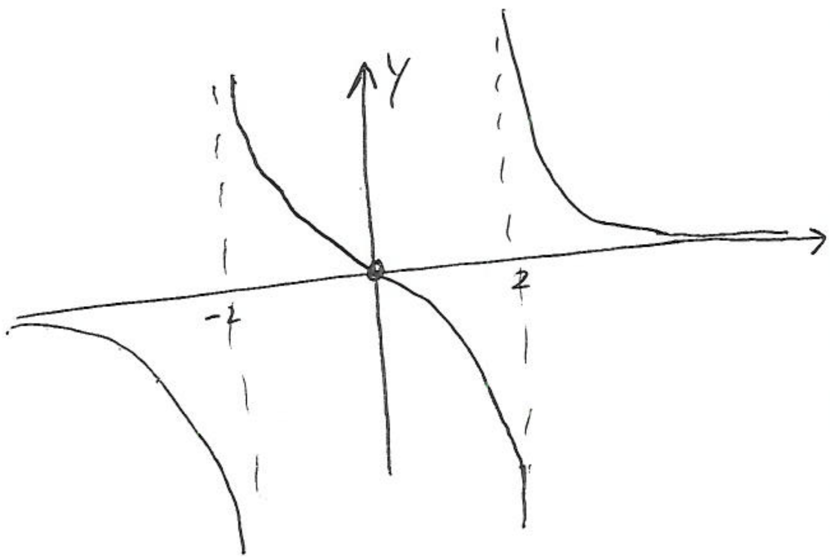
$\lim_{x \to \infty} f = 0 \quad \lim_{x \to -\infty} f = 0$

$\lim_{x \to 2^+} f = +\infty \quad \lim_{x \to 2^-} f = -\infty$

$\lim_{x \to -2^+} f = +\infty \quad \lim_{x \to -2^-} f = -\infty$

$y=0$ is a Horizontal asymptote
$x=2$ and $x=-2$ are Vertical asys.

f is c.u for $2<x$ also for $-2<x<0$
f is c.d. for $x<-2$ also for $0<x<2$
$(0,0)$ is the point of inflection



$f' = \frac{(x^2-4) - x\,2x}{(x^2-4)^2} = \frac{-x^2-4}{(x^2-4)^2}$

$= \frac{-(x^2+4)}{(x^2-4)^2}$ always negative

f always decreasing
f is never increasing
no local max pts no local min points.

$f' = -(x^2+4)(x^2-4)^{-2}$

$f'' = -(x^2+4)(-2)(x^2-4)^{-3}\,2x - 2x(x^2-4)^{-2}$

$= \frac{2x\left((x^2+4)2 - (x^2-4)\right)}{(x^2-4)^3} = \frac{2x(x^2+12)}{(x^2-4)^3}$

f'' neg $\mid_{-2}$ f'' pos $\mid_{0}$ f'' neg $\mid_{2}$ f'' pos

6. State the Mean Value Theorem.

If $f(x)$ is differentiable for $a \le x \le b$, then there exists a number c with $f'(c) = \frac{f(b)-f(a)}{b-a}$ and $a \le c \le b$.

7. Each side of a square is growing at the rate of 4 inches per second? How fast is the area of the square growing, when the length of each side is 10 inches?

$\frac{d\ell}{dt} = 4$

$\left.\frac{dA}{dt}\right|_{\ell=10}$

$A = \ell^2$

$\frac{dA}{dt} = 2\ell \frac{d\ell}{dt}$

$\left.\frac{dA}{dt}\right|_{\ell=10} = 2(10)\cdot 4 \ \frac{in^2}{sec}$

8. The height of an object above the ground at time t is $s(t) = -16t^2 + 32t + 48$, where s is measured in feet and t is measured in seconds. What is the velocity of the object when it strikes the ground?

The object hits the ground when

$-16(t^2 - 2t - 3) = 0$

$-16(t-3)(t+1) = 0$

so $t = 3$ or $t = -1$

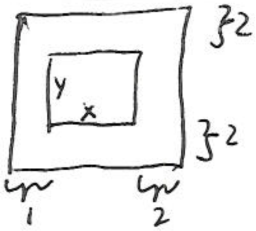
not interesting to us

so the object hits the ground at time $t = 3$

$V(t) = -32t + 32$

$V(3) = -32(3) + 32 \ \frac{ft}{sec}$

9. A page of a book is to contain 27 square inches of print. If the margins at the top, bottom, and one side are 2 inches and the margin at the other side is 1 inch, what size page would use the least paper?



$xy = 27$

Minimize $A = (x+3)(y+4)$

Minimize $A = (x+3)\left(\frac{27}{x} + 4\right)$

$A = 27 + \frac{81}{x} + 4x + 12$

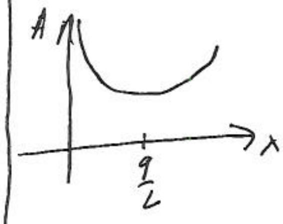
$A' = \frac{-81}{x^2} + 4$

$A' = 0$ when $\frac{81}{x^2} = 4$

$\frac{81}{4} = x^2$

$\frac{9}{2} = x$

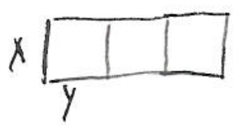
(we care only about $0 < x$.)



when $x = \frac{9}{2}$ $y = 6$

The area is minimized when the paper is $\frac{9}{2} + 3$ inches by $6 + 4$ inches

10. A farmer wishes to fence off three identical adjoining rectangular pens, each with 300 square feet of area. What should the width and length of each pen be so that the least amount of fence is required?



$xy = 300$ $\quad y = \frac{300}{x}$

$F = 4x + 6y$

$F = 4x + \frac{1800}{x}$ $\quad 0 < x$

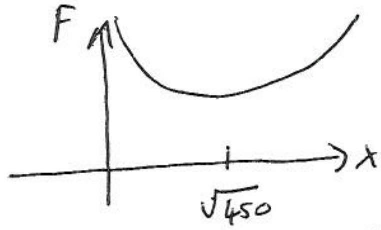
Minimize F

$F' = 4 - \frac{1800}{x^2}$

$F' = 0$ when $\frac{4x^2 - 1800}{x^2} = 0$

so $x^2 = \frac{1800}{4} = 450$

so $x = \sqrt{450}$ (our $x > 0$)



To minimize the amt of fence take $x = \sqrt{450}$ ft and $y = \frac{300}{\sqrt{450}}$ ft