1996 Exam 2 38

PRINT Your Name:____________________________ Section:__________

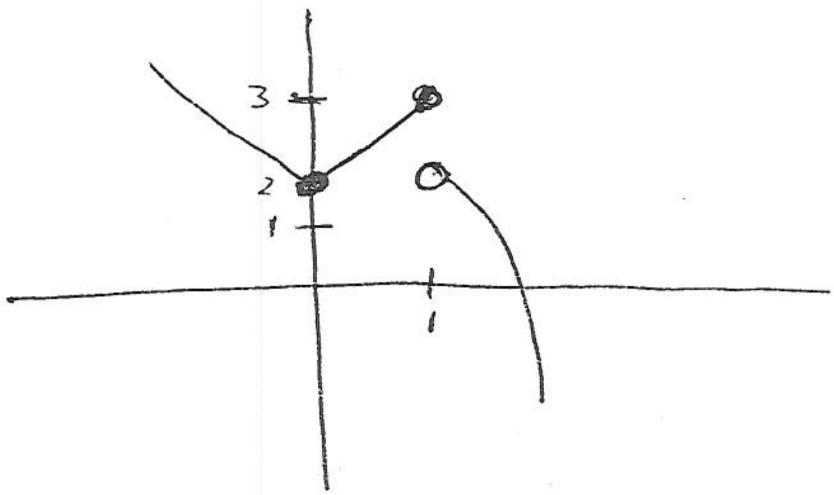
There are 10 problems on 5 pages. Each problems is worth 10 point. In problem 2 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer.

NO CALCULATORS!

1. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 2-x & \text{if } x<0, \\ 2+x & \text{if } 0 \le x \le 1, \text{ and} \\ 3-x^2 & \text{if } 1 < x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = \underline{2}$ $\quad \lim_{x\to 0+} f(x) = \underline{2}$ $\quad \lim_{x\to 0-} f(x) = \underline{2}$ $\quad \lim_{x\to 0} f(x) = \underline{2}$

$f(1) = \underline{3}$ $\quad \lim_{x\to 1+} f(x) = \underline{2}$ $\quad \lim_{x\to 1-} f(x) = \underline{3}$ $\quad \lim_{x\to 1} f(x) = \underline{DNE}$

$f(2) = \underline{-1}$ $\quad \lim_{x\to 2+} f(x) = \underline{-1}$ $\quad \lim_{x\to 2-} f(x) = \underline{-1}$ $\quad \lim_{x\to 2} f(x) = \underline{-1}$

(c) Where is $f(x)$ continuous?

Everywhere except $x = 1$.

(d) Where is $f(x)$ differentiable?

Everywhere except $x = 0$ and $x = 1$.

1996 Ex2 39

2. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = 4\sqrt{2x-3}$.

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{4\sqrt{2(x+h)-3} - 4\sqrt{2x-3}}{h}$$

$$= \lim_{h \to 0} \frac{4}{h} \frac{\left(\sqrt{2x+2h-3} - \sqrt{2x-3}\right)\left(\sqrt{2x+2h-3} - \sqrt{2x-3}\right)}{\left(\sqrt{2x+2h-3} + \sqrt{2x-3}\right)}$$

$$= \lim_{h \to 0} \frac{4}{h} \frac{\cancel{2x} + 2h \cancel{-3} - (\cancel{2x} \cancel{-3})}{\left(\sqrt{2x+2h-3} + \sqrt{2x-3}\right)} = \lim_{h \to 0} \frac{8\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h-3} + \sqrt{2x-3}\right)}$$

$$= \frac{8}{2\sqrt{2x-3}} = \frac{4}{\sqrt{2x-3}}$$

3. Find the equation of the line tangent to $f(x) = x^5 - 3x^2$ at $x = 2$.

$f(2) = 32 - 12 = 20$

$f'(x) = 5x^4 - 6x$

$f'(2) = 80 - 12 = 68$

$$\boxed{y - 20 = 68(x-2)}$$

4. The position of an object above the surface of the earth is given by $s(t) = -16t^2 + 64t + 100$, where s is measured in feet and t is measured in seconds. How high does the object get?

$s' = -32t + 64$

$s' = 0$ when $2 = t$

Max ht $= s(2) = -16(4) + 64(2) + 100 = 164$ ft

1996 ex 2 40

5. Let $y = x^2 \cos^2(4x^5 + 19x)$. Find dy.

$dy = \frac{dy}{dx}\,dx = \left[x^2\, 2\cos(4x^5+19x)\left[-\sin(4x^5+19x)\right]\right](20x^4+19) + 2x\cos^2(4x^5+19x$

$dy = \left[-2x^2\cos(4x^5+19x)\sin(4x^5+19x)\ (20x^4+19) + 2x\cos^2(4x^5+19x)\right] dx$

6. Let $y = \sqrt{4x^3 + 9x + \sin^3(\cos(5x^4 + 3x))}$. Find $\frac{dy}{dx}$.

$y' = \dfrac{12x^2+9 + 3\sin^2(\cos(5x^4+3x))\cos(\cos(5x^4+3x))\left[-\sin(5x^4+3x)\right](20x^3+3)}{2\sqrt{4x^3+9x+\sin^3(\cos(5x^4+3x))}}$

1996 Ex 2

41

7. Let $3x^2y^3 = \sin(xy^2) + 3x^5$. Find $\frac{dy}{dx}$.

$$9x^2y^2\frac{dy}{dx} + 6xy^3 = \cos(xy^2)\left[2xy\frac{dy}{dx} + y^2\right] + 15x^4$$

$$\frac{dy}{dx}\left[9x^2y^2 - 2xy\cos(xy^2)\right] = y^2\cos(xy^2) + 15x^4 - 6xy^3$$

$$\boxed{\frac{dy}{dx} = \frac{y^2\cos(xy^2) + 15x^4 - 6xy^3}{9x^2y^2 - 2xy\cos(xy^2)}}$$

8. Let $y = \frac{3}{x} + 15 - 4\sqrt{x}$. Find $\frac{d^2y}{dx^2}$.

$$y' = -3x^{-2} - 2x^{-\frac{1}{2}}$$

$$\boxed{y'' = 6x^{-3} + x^{-\frac{3}{2}}}$$

1996 Ex2

42

9. The area of a square is growing at the rate of 4 square feet per second. How fast is the length of each side growing when each side has length 6 feet?

Let A = the area of the square at time t

s = the length of each side at time t

We know $\frac{dA}{dt} = 4\ ft^2/sec$

We want $\left.\frac{ds}{dt}\right|_{s=6\,ft}$

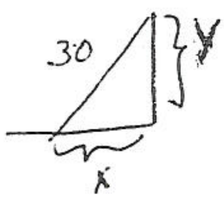
$$A = s^2$$

$$\frac{dA}{dt} = 2s\frac{ds}{dt}$$

$$\frac{ds}{dt} = \frac{1}{2s}\frac{dA}{dt}$$

$$\left.\frac{ds}{dt}\right|_{s=6\,in} = \frac{1}{12\,ft}\, 4\,\frac{ft^2}{sec} = \frac{1}{3}\,\frac{ft}{sec.}$$

10. A 30 foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 3 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 5 feet from the wall?



Let X be the distance from the base of the ladder to the wall

Let Y be the distance from the top of the ladder to the wall.

We know $\frac{dX}{dt} = 3\ ft/sec.$ We want $\left.\frac{dY}{dt}\right|_{X=5}$

$$30^2 = X^2 + Y^2$$

$$0 = 2X\frac{dX}{dt} + 2Y\frac{dY}{dt}$$

$$-\frac{X}{Y}\frac{dX}{dt} = \frac{dY}{dt}$$

$$\left.\frac{dy}{dt}\right|_{X=5} = \frac{-5}{\sqrt{900-25}}\, 3\,\frac{ft}{sec.}$$