Exam 2 141 2000

PRINT Your Name:____________________ Recitation Time ________ Tu. Th.

There are 10 problems on 6 pages. Each problem is worth 10 points. In problem 3 you MUST use the definition of the derivative. In the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!**

1. Let $y = x\sin x$. Find dy.

$dy = (x\cos x + \sin x)\,dx$

2. Let $y = \sin\left(x^3\cos^2(2x) + 19x^2\right)$. Find $\frac{dy}{dx}$.

$\frac{dy}{dx} = \cos\left(x^3\cos^2(2x) + 19x^2\right)\left[-x^3 2\cos 2x(\sin 2x)2 + 3x^2\cos^2 2x + 38x\right]$

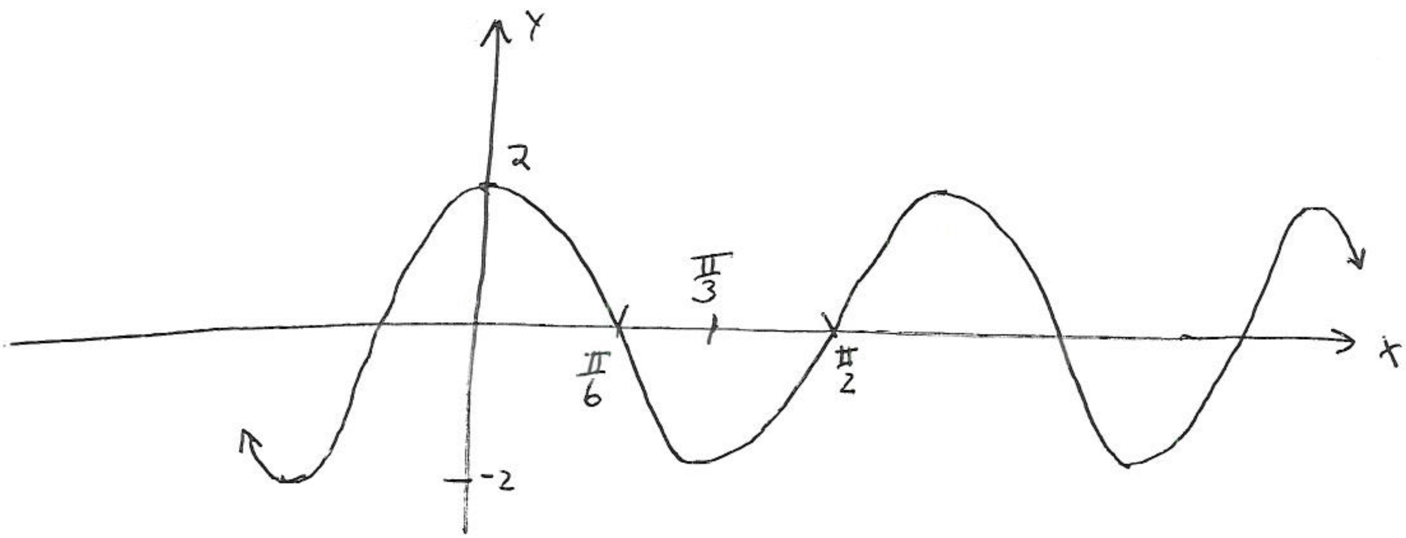
3. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \dfrac{2}{3x-4}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h \to 0} \frac{\frac{2}{3(x+h)-4} - \frac{2}{3x-4}}{h} = \lim_{h \to 0} \frac{2(3x-4)-2(3x+3h-4)}{h(3x-4)(3x+3h-4)}$$

$$= \lim_{h \to 0} \frac{\cancel{6x}-\cancel{8}-\cancel{6x}-6h+\cancel{8}}{h(3x-4)(3x+3h-4)} = \lim_{h \to 0} \frac{-6h}{h(3x-4)(3x+3h-4)}$$

$$= \lim_{h \to 0} \frac{-6}{(3x-4)(3x+3h-4)} = \frac{-6}{(3x-4)(3x-4)} = \frac{-6}{(3x-4)^2}$$

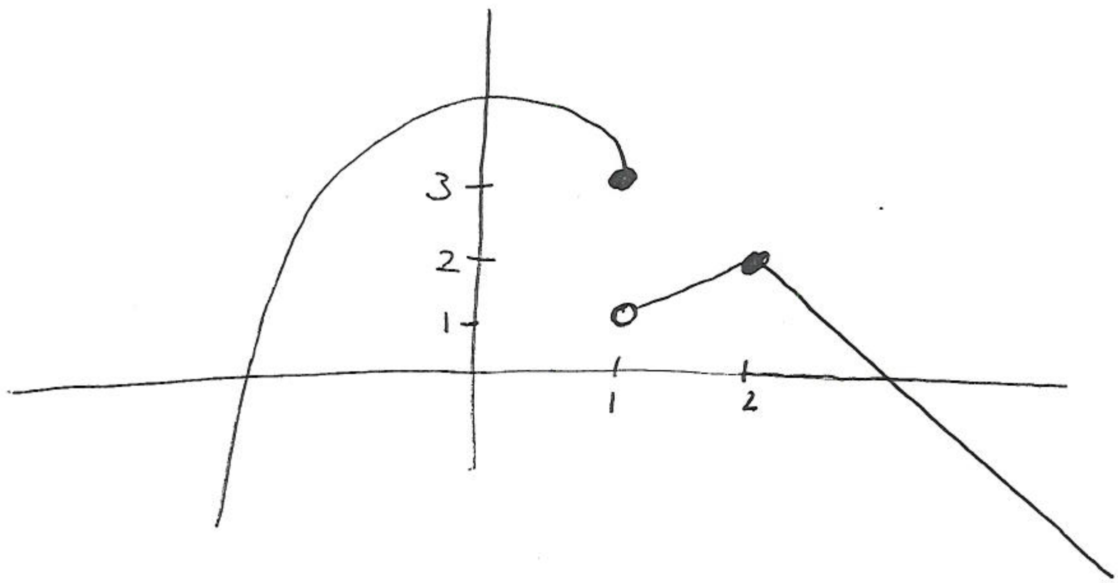
4. Graph $y = 2\cos 3x$. Mark a few points on each axis.



5. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 4 - x & \text{if } 2 \le x, \\ x & \text{if } 1 < x < 2, \text{ and} \\ 4 - x^2 & \text{if } x \le 1. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = \underline{4}$ $\quad \lim_{x \to 0^+} f(x) = \underline{4}$ $\quad \lim_{x \to 0^-} f(x) = \underline{4}$ $\quad \lim_{x \to 0} f(x) = \underline{4}$

$f(1) = \underline{3}$ $\quad \lim_{x \to 1^+} f(x) = \underline{1}$ $\quad \lim_{x \to 1^-} f(x) = \underline{3}$ $\quad \lim_{x \to 1} f(x) = \underline{DNE}$

$f(2) = \underline{2}$ $\quad \lim_{x \to 2^+} f(x) = \underline{2}$ $\quad \lim_{x \to 2^-} f(x) = \underline{2}$ $\quad \lim_{x \to 2} f(x) = \underline{2}$

(c) Where is $f(x)$ continuous?

f(x) is continuous for all x except x = 1.

(d) Where is $f(x)$ differentiable?

f(x) is differentiable for all x except x = 1, 2

6. The volume of a cube is growing at the rate of 6 cubic inches per second. Find the rate at which each side of the cube is growing at the instant when each side has length 10 inches.

$$\frac{dV}{dt} = 6 \text{ in}^3/\text{sec}$$

$$\left.\frac{d\ell}{dt}\right|_{\ell=10\text{ in}} = ?$$

V = volume of cube
ℓ = length of each side

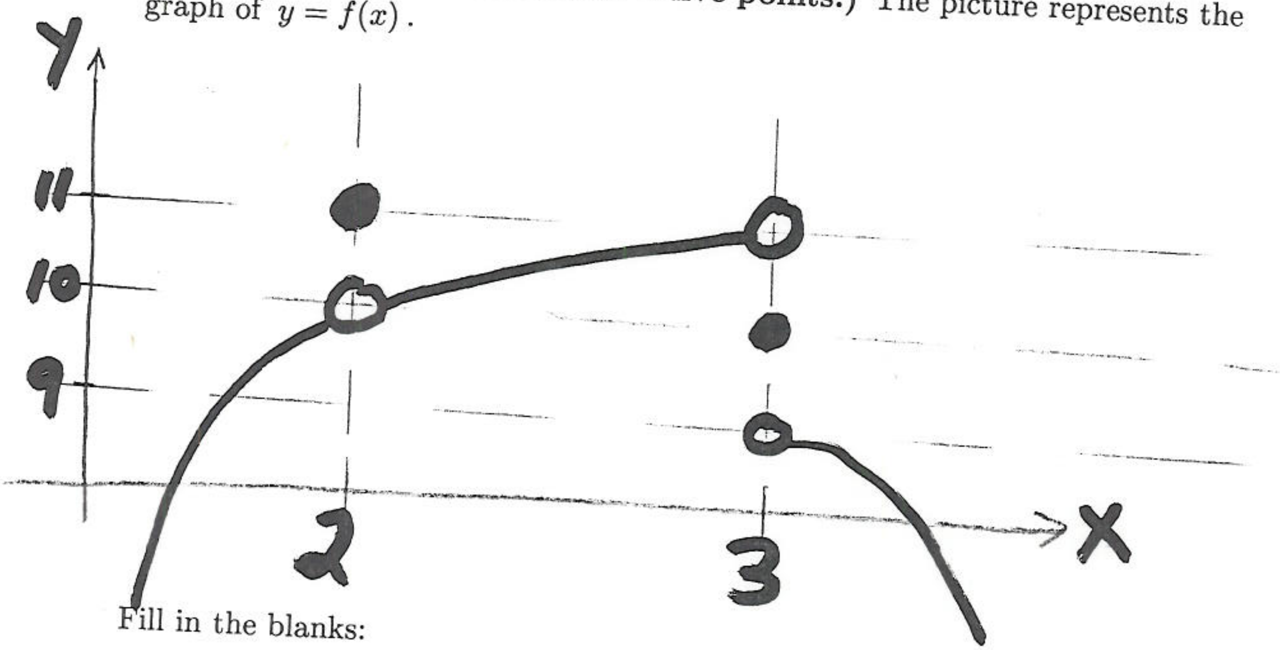
$$V = \ell^3$$

$$\frac{dV}{dt} = 3\ell^2 \frac{d\ell}{dt}$$

$$\frac{d\ell}{dt} = \frac{1}{3\ell^2}\frac{dV}{dt}$$

$$\left.\frac{d\ell}{dt}\right|_{\ell=10\text{ in}} = \frac{1}{3(10\text{ in})^2}\, 6\,\frac{\text{in}^3}{\text{sec}} = \frac{6}{300}\,\frac{\text{in}}{\text{sec}} = \frac{1}{50}\,\frac{\text{in}}{\text{sec}}$$

7. **(The penalty for each mistake is five points.)** The picture represents the graph of $y = f(x)$.



Fill in the blanks:

$f(2) = \underline{11}$ $\quad \lim_{x\to 2^+} f(x) = \underline{10}$ $\quad \lim_{x\to 2^-} f(x) = \underline{10}$ $\quad \lim_{x\to 2} f(x) = \underline{10}$

$f(3) = \underline{10}$ $\quad \lim_{x\to 3^+} f(x) = \underline{9}$ $\quad \lim_{x\to 3^-} f(x) = \underline{11}$ $\quad \lim_{x\to 3} f(x) = \underline{DNE}$

8. Let $4x^5y^3 = \sin(3x^4y^6)$. Find $\frac{dy}{dx}$.

$$4x^5 3y^2 \frac{dy}{dx} + 20x^4y^3 = \cos(3x^4y^6)\left[18x^4y^5\frac{dy}{dx} + 12x^3y^6\right]$$

$$\frac{dy}{dx}\left[12x^5y^2 - 18x^4y^5\cos(3x^4y^6)\right] = +12x^3y^6\cos(3x^4y^6) - 20x^4y^3$$

$$\frac{dy}{dx} = \frac{12x^3y^6\cos(3x^4y^6) - 20x^4y^3}{12x^5y^2 - 18x^4y^5\cos(3x^4y^6)}$$

9. Find the equation of the line tangent to $f(x) = \cos^2 x$ at $x = \frac{\pi}{4}$.

The y-coordinate is $f(\frac{\pi}{4}) = (\cos\frac{\pi}{4})^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{2}{4} = \frac{1}{2}$

$f'(x) = -2\cos x \sin x$

$f'(\frac{\pi}{4}) = -\cancel{2}\frac{\sqrt{2}}{\cancel{2}}\frac{\sqrt{2}}{2} = -1$

line: $(y - \frac{1}{2}) = -1(x - \frac{\pi}{4})$

10. The height of an object above the ground at time t is $s(t) = -16t^2 + 32t + 48$, where s is measured in feet and t is measured in seconds. What is the velocity of the object when it strikes the ground?

The object strikes the ground when $s(t) = 0$

$-16(t^2 - 2t - 3) = 0$

$-16(t-3)(t+1) = 0$

$t = 3$ or -1

i.e. $t = 3$

$v(t) = s'(t) = -32t + 32$

$V(3) = -32(3) + 32 = -32(3-1) = -32(2) = -64$ ft/sec