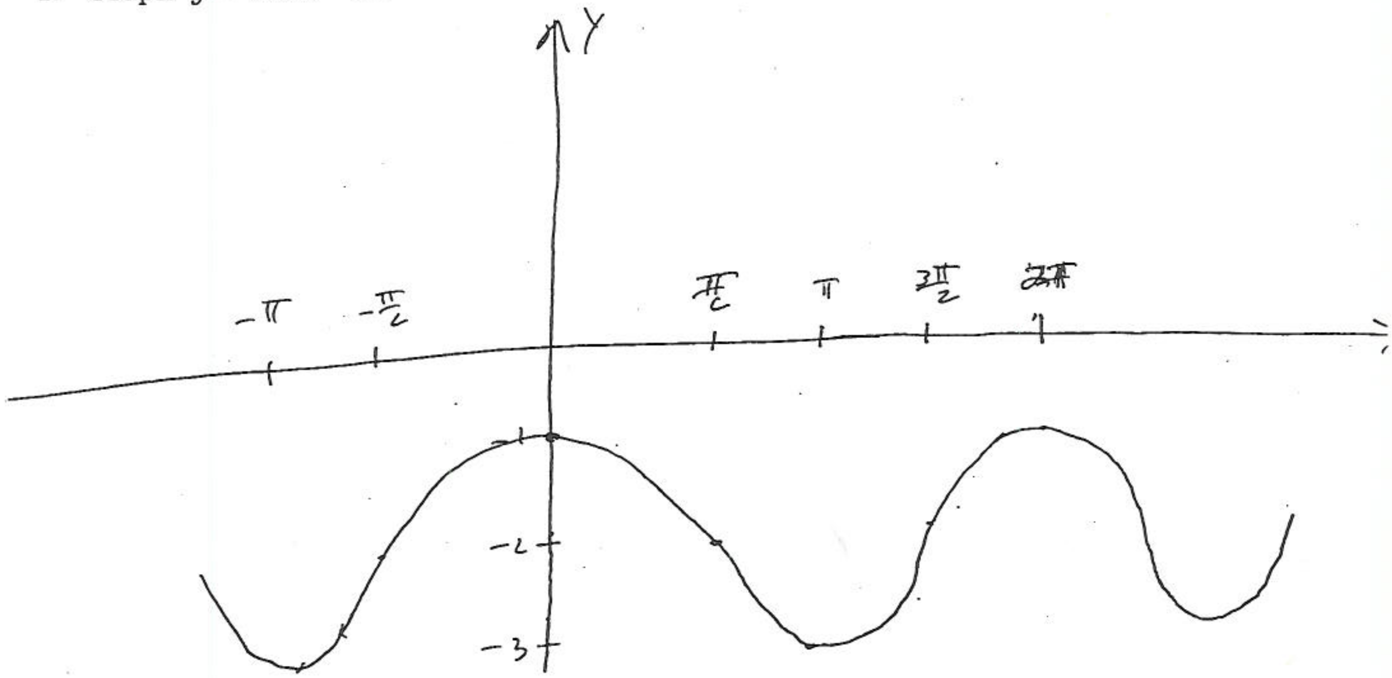
1999 Exam 1

65

PRINT Your Name:____________________________ Section:__________

There are 9 problems on 4 pages. Each problem, unless otherwise noted, is worth 10 points. In one problem you are instructed to use the definition of the derivative; you MUST use the definition of the derivative in that problem. In the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Graph $y = \cos x - 2$.



2. Let $f(x) = 9x^4 + \dfrac{8}{x} + 3\sqrt{x} + 6$. Find $f'(x)$.

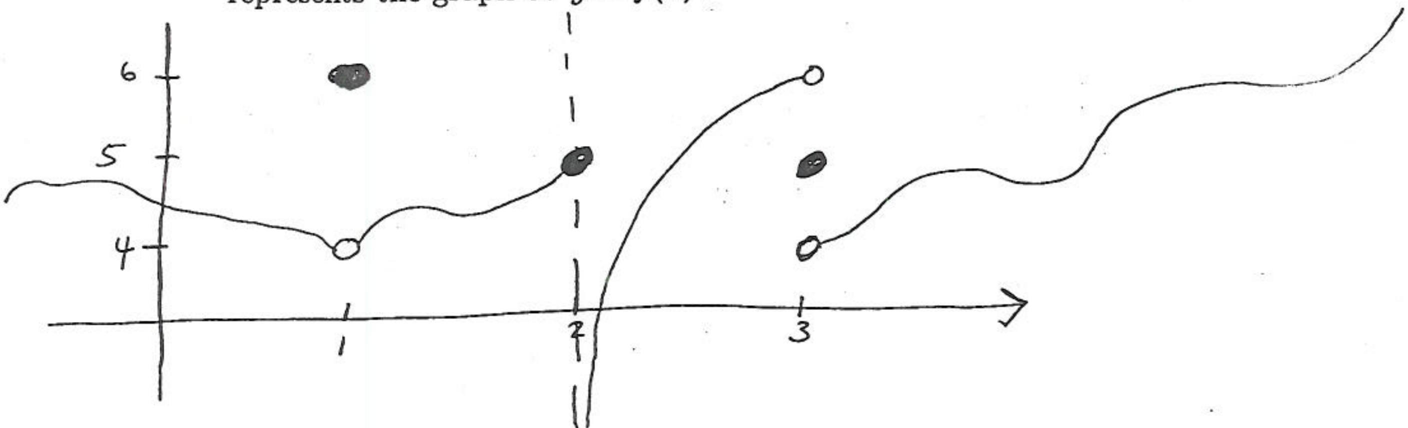
$f(x) = 9x^4 + 8x^{-1} + 3x^{\frac{1}{2}} + 6$

$f'(x) = 36x^3 - 8x^{-2} + \frac{3}{2}x^{-\frac{1}{2}}$

1999 Exam 1

66

3. (14 points) **(The penalty for each mistake is four points.)** The picture represents the graph of $y = f(x)$.



(a) Fill in the blanks:

$f(1) = 6$	$\lim_{x\to 1^+} f(x) = 4$	$\lim_{x\to 1^-} f(x) = 4$	$\lim_{x\to 1} f(x) = 4$
$f(2) = 5$	$\lim_{x\to 2^+} f(x) = -\infty$	$\lim_{x\to 2^-} f(x) = 5$	$\lim_{x\to 2} f(x) =$ DNE
$f(3) = 5$	$\lim_{x\to 3^+} f(x) = 4$	$\lim_{x\to 3^-} f(x) = 6$	$\lim_{x\to 3} f(x) =$ DNE

(b) Where is f continuous?

everywhere except $x=1, 2, 3$

(c) Where is f differentiable?

everywhere except $x=1, 2, 3$

4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \frac{4}{x} - 3$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{4}{x+h} - 3 - \left(\frac{4}{x} - 3\right)}{h} =$$

$$\lim_{h\to 0} \frac{\frac{4}{x+h} - \frac{4}{x}}{h} = \lim_{h\to 0} \frac{4x - 4(x+h)}{h(x+h)(x)} = \lim_{h\to 0} \frac{\cancel{4x} - \cancel{4x} - 4h}{h(x+h)x}$$

$$= \lim_{h\to 0} \frac{-4\cancel{h}}{\cancel{h}(x+h)x} = \lim_{h\to 0} \frac{-4}{(x+h)x} = -\frac{4}{x^2}$$

1999 Exam 1

67

5. Let $f(x) = (x+6)\sqrt{x}$. Find $f'(x)$.

$f(x) = x^{\frac{3}{2}} + 6x^{\frac{1}{2}}$

$f'(x) = \frac{3}{2}x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$

6. Find the equation of the line tangent to $f(x) = 10x^{11} + 12x$ at $x = -1$.

$f'(x) = 110x^{10} + 12$

$f'(-1) = 110 + 12 = 122$

$f(-1) = -10 - 12 = -22$

$y - y_0 = m(x - x_0)$

$\boxed{y + 22 = 122(x+1)}$

7. Find $\lim_{x\to 0} \frac{1-\cos x}{x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)}$

$= \lim_{x\to 0} \frac{\sin^2 x}{x^2(1+\cos x)} = \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{x} \cdot \frac{1}{1+\cos x} = 1 \cdot 1 \cdot \frac{1}{2}$

$= \frac{1}{2}$

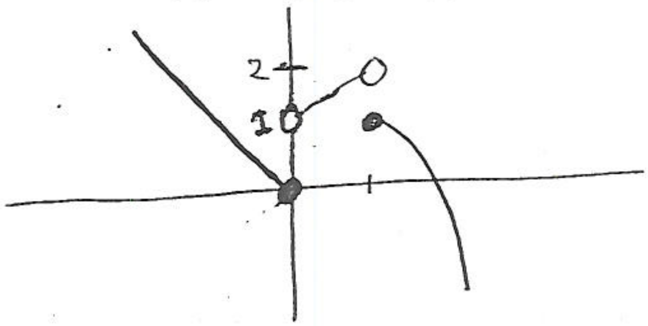
1999 Exam 1

68

8. (14 points) (The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} 2 - x^2 & \text{if } 1 \leq x, \\ x + 1 & \text{if } 0 < x < 1, \text{ and} \\ -x & \text{if } x \leq 0. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = 0$	$\lim\limits_{x\to0^+} f(x) = 1$	$\lim\limits_{x\to0^-} f(x) = 0$	$\lim\limits_{x\to0} f(x) = DNE$
$f(1) = 1$	$\lim\limits_{x\to1^+} f(x) = 1$	$\lim\limits_{x\to1^-} f(x) = 2$	$\lim\limits_{x\to1} f(x) = DNE$
$f(2) = -2$	$\lim\limits_{x\to2^+} f(x) = -2$	$\lim\limits_{x\to2^-} f(x) = -2$	$\lim\limits_{x\to2} f(x) = -2$

(c) Where is $f(x)$ continuous?

f is continuous everywhere except at $X = 0$ and $X = 1$

(d) Where is $f(x)$ differentiable?

f is differentiable everywhere except at $x=0$ and $x=1$

9. (12 points – 3 points for each part) Compute the following limits:

(a) $\lim\limits_{x\to3^+} \dfrac{x^2 - x - 6}{x - 3} = \lim\limits_{x\to3^+} \dfrac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}} = 5$

(c) $\lim\limits_{x\to3^+} \dfrac{x - 3}{x^2 - x - 6} \quad \lim\limits_{x\to3^+} \dfrac{x-3}{(x-3)(x+2)} = \dfrac{1}{5}$

(c) $\lim\limits_{x\to3^+} \dfrac{x^2 - x - 6}{x + 3} = 0$

(d) $\lim\limits_{x\to3^+} \dfrac{x + 3}{x^2 - x - 6} = +\infty$