1995 Exam 1 (1)

PRINT Your Name:_____

There are 13 problems on 4 pages. Problem 2 is worth 12 points; problem 3 is worth 11 points; each of the other problems is worth 7 points. In problem 7 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. CIRCLE your answer.

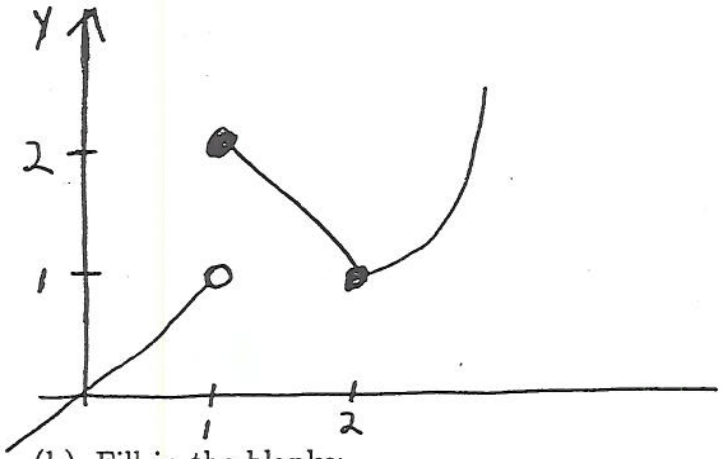
1. Let $f(x) = 2x + 1$ and $g(x) = 3x^2$.
   (a) Find $(f \circ g)(x)$. $= f(3x^2) = 2(3x^2) + 1 = 6x^2 + 1$

   (b) Find $(g \circ f)(x) = g(2x+1) = 3(2x+1)^2$

2. (The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} x & \text{if } x < 1, \\ 3 - x & \text{if } 1 \le x \le 2, \text{ and} \\ x^2 - 3 & \text{if } 2 < x. \end{cases}$$

   (a) Graph $y = f(x)$.

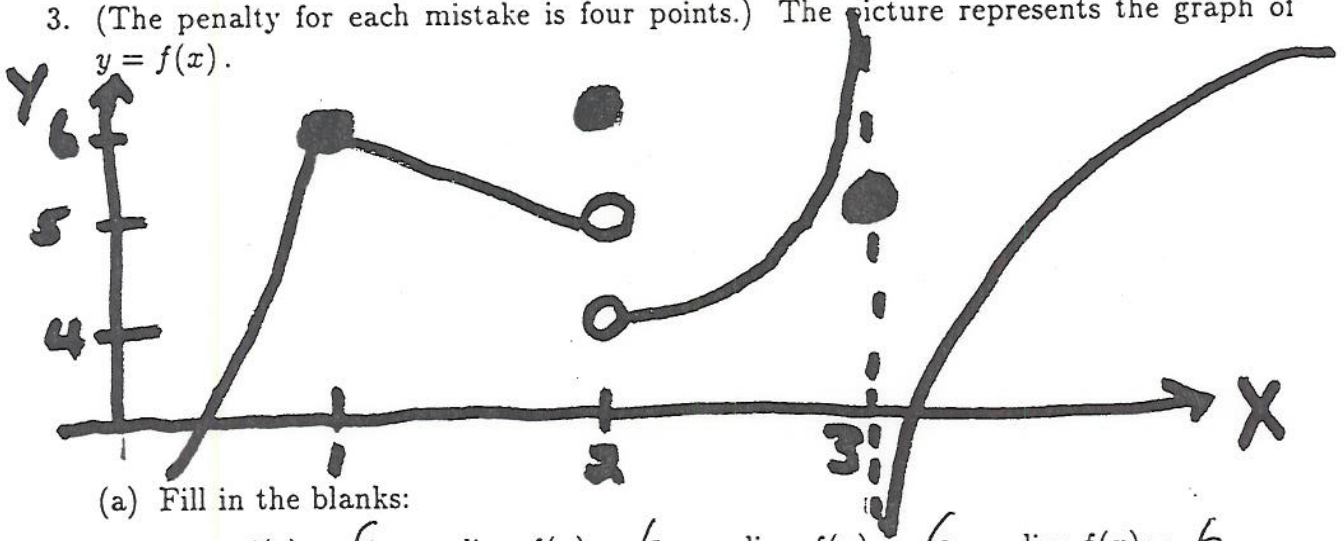


   (b) Fill in the blanks:

$f(1) = 2$	$\lim_{x \to 1^+} f(x) = 2$	$\lim_{x \to 1^-} f(x) = 1$	$\lim_{x \to 1} f(x) = DNE$
$f(2) = 1$	$\lim_{x \to 2^+} f(x) = 1$	$\lim_{x \to 2^-} f(x) = 1$	$\lim_{x \to 2} f(x) = 1$
$f(3) = 6$	$\lim_{x \to 3^+} f(x) = 6$	$\lim_{x \to 3^-} f(x) = 6$	$\lim_{x \to 3} f(x) = 6$

1995 Ex 1 (2)

3. (The penalty for each mistake is four points.) The picture represents the graph of $y = f(x)$.



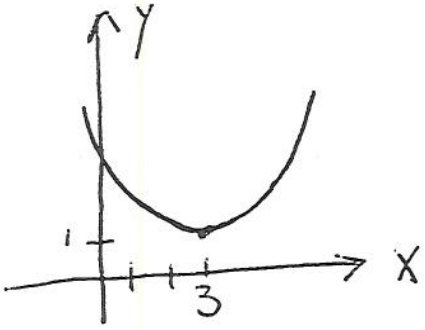
(a) Fill in the blanks:

$f(1) = \underline{6}$	$\lim_{x\to 1^+} f(x) = \underline{6}$	$\lim_{x\to 1^-} f(x) = \underline{6}$	$\lim_{x\to 1} f(x) = \underline{6}$
$f(2) = \underline{6}$	$\lim_{x\to 2^+} f(x) = \underline{4}$	$\lim_{x\to 2^-} f(x) = \underline{5}$	$\lim_{x\to 2} f(x) = \underline{DNE}$
$f(3) = \underline{5}$	$\lim_{x\to 3^+} f(x) = \underline{-\infty}$	$\lim_{x\to 3^-} f(x) = \underline{+\infty}$	$\lim_{x\to 3} f(x) = \underline{DNE}$

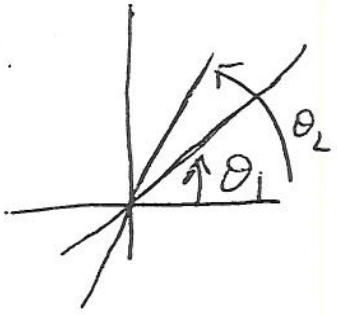
(b) Where is f discontinuous? at $x = 2, 3$

(c) Where is f not differentiable? at $x = 1, 2, 3$

4. Graph $y = (x-3)^2 + 1$.



5. Find the angle (in radians) between $y = x$ and $y = 2x$.



$\theta_1 = \arctan 1 \cong .7853982$ radians

$\theta_2 = \arctan 2 \cong 1.1071487$ radians

$\text{ans} = \theta_2 - \theta_1 \cong .3217506$ radians

1995 Ex 1

(3)

6. Express $\sin(x-y)$ in terms of $\sin x$, $\sin y$, $\cos x$, and $\cos y$.

$\sin(x-y) = \sin x \cos y - \cos x \sin y$

7. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \sqrt{x}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h}-\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})} = \lim_{h\to 0} \frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{\sqrt{x}+\sqrt{x}} = \frac{1}{2\sqrt{x}}$$

8. Find the equation of the line tangent to $f(x) = 4x^4 + 3x^2$ at the point $(1, 7)$.

$f'(x) = 16x^3 + 6x$

$f'(1) = 16+6 = 22$

$y - 7 = 22(x-1)$

1995 Ex 1

(4)

9. Find $\lim_{x\to 0} \frac{\sin x}{x^3}$. $= \lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{x^2} = 1 \cdot +\infty = +\infty$

10. Find $\lim_{x\to 1} \frac{x^2-3x+2}{x-1}$. $= \lim_{x\to 1} \frac{(x-1)(x-2)}{x-1} = \lim_{x\to 1} x-2 = -1$

11. Find $\lim_{x\to 1} \frac{x^2-3x+2}{(x+1)^2} = \frac{0}{2} = 0$

12. Find $\lim_{x\to 1} \frac{x^2+3x+2}{(x-1)^2}$. $= +\infty$

top goes to 6
bottom goes to 0 and bottom is positive

13. Let $f(x) = (2x^3+3x)(8x^2+19x+1)$. Find $f'(x)$.

$f'(x) = (2x^3+3x)(16x+19) + (8x^2+19x+1)(6x^2+3)$

1995 Exam 2

5

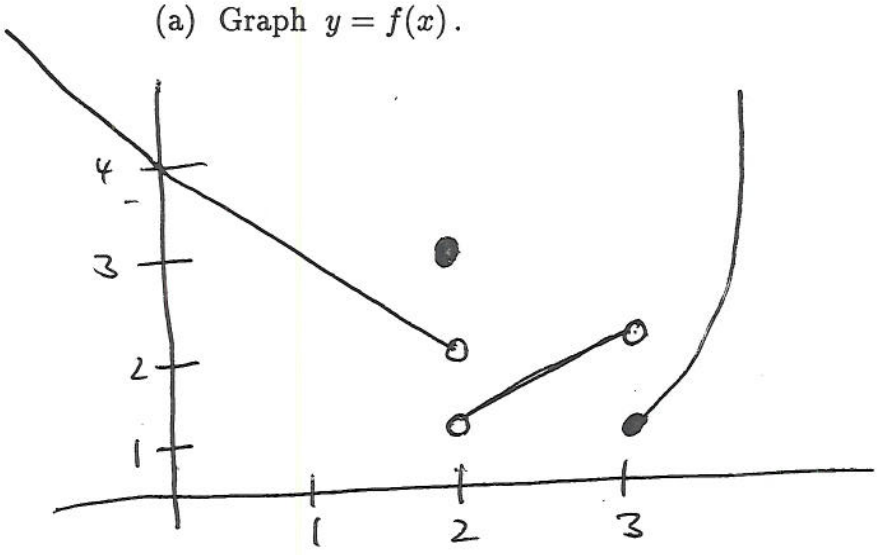
PRINT Your Name:_____

There are 13 problems on 6 pages. In problem 10 you MUST use the definition of the derivative; in the other problems you may use any legitimate derivative rule. SHOW your work. CIRCLE your answer.

1. (10 points – The penalty for each mistake is four points.) Let

$$f(x) = \begin{cases} 4 - x & \text{if } x < 2, \\ 3 & \text{if } x = 2, \\ x - 1 & \text{if } 2 < x < 3, \text{ and} \\ x^2 - 8 & \text{if } 3 \leq x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(1) = \underline{3}$	$\lim_{x\to 1^+} f(x) = \underline{3}$	$\lim_{x\to 1^-} f(x) = \underline{3}$	$\lim_{x\to 1} f(x) = \underline{3}$
$f(2) = \underline{3}$	$\lim_{x\to 2^+} f(x) = \underline{1}$	$\lim_{x\to 2^-} f(x) = \underline{2}$	$\lim_{x\to 2} f(x) = \underline{DNE}$
$f(3) = \underline{1}$	$\lim_{x\to 3^+} f(x) = \underline{1}$	$\lim_{x\to 3^-} f(x) = \underline{2}$	$\lim_{x\to 3} f(x) = \underline{DNE}$

2. (7 points) Let $y = \frac{1}{\sqrt{2x}} - \sin(2x)$. Find $\frac{dy}{dx}$.

$y = \frac{1}{\sqrt{2}} x^{-\frac{1}{2}} - \sin(2x)$

$\frac{dy}{dx} = \frac{1}{\sqrt{2}}\left(-\frac{1}{2}\right) x^{-\frac{3}{2}} - 2\cos(2x)$

$-(2x)^{-\frac{3}{2}} - 2\cos(2x)$