Exam 1 141 2000

PRINT Your Name:______________________________ Section:__________

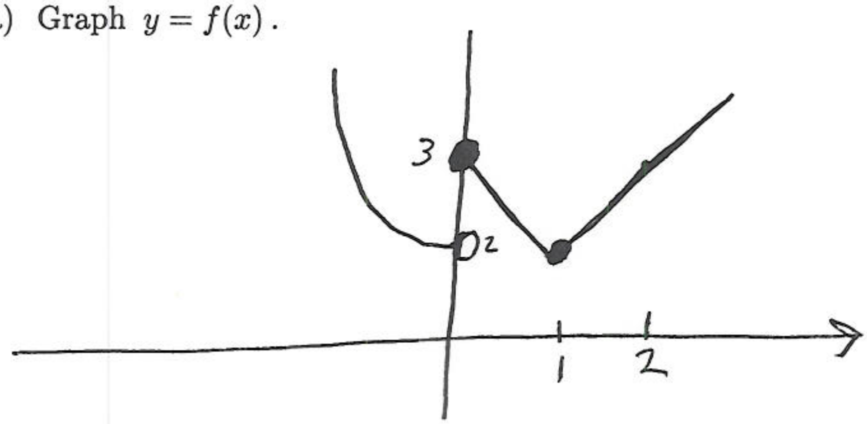
There are 8 problems on 4 pages. Problems 1 and 5 and are worth 15 points each. Problem 8 is worth 20 points. The other problems are worth 10 points each. In problem 3 you MUST use the definition of the derivative. SHOW your work. $\boxed{CIRCLE}$ your answer.

NO CALCULATORS!

1. **(The penalty for each mistake is four points.)** Let

$$f(x) = \begin{cases} x^2+2 & \text{if } x<0, \\ 3-x & \text{if } 0 \le x \le 1, \text{ and} \\ x+1 & \text{if } 1 < x. \end{cases}$$

(a) Graph $y = f(x)$.



(b) Fill in the blanks:

$f(0) = 3$	$\lim_{x\to 0+} f(x) = 3$	$\lim_{x\to 0^-} f(x) = 2$	$\lim_{x\to 0} f(x) =$ DNE
$f(1) = 2$	$\lim_{x\to 1+} f(x) = 2$	$\lim_{x\to 1^-} f(x) = 2$	$\lim_{x\to 1} f(x) = 2$
$f(2) = 3$	$\lim_{x\to 2+} f(x) = 3$	$\lim_{x\to 2^-} f(x) = 3$	$\lim_{x\to 2} f(x) = 3$

(c) Where is $f(x)$ continuous?

f(x) is continuous for all X except $X = 0$

(d) Where is $f(x)$ differentiable?

f(x) is differentiable for all X except $X=0$ and $X=1$.

2. Let $f(x) = 3x-1$ and $g(x) = 2x^2+1$.

(a) Find $(f \circ g)(x)$.

$f(g(x)) = f(2x^2+1) = 3(2x^2+1)-1$

(b) Find $(g \circ f)(x)$.

$g(f(x)) = g(3x-1) = 2(3x-1)^2+1$

3. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = 3\sqrt{x-4}$.

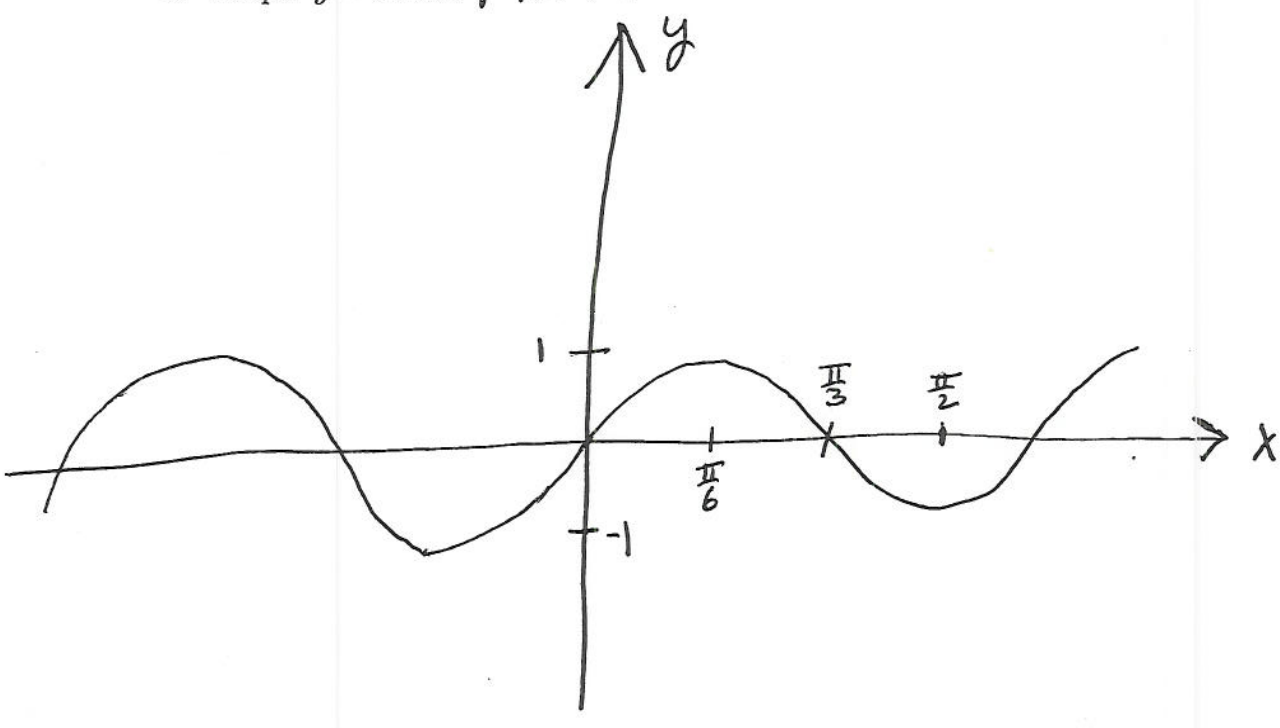
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(h)}{h} = \lim_{h \to 0} \frac{3\sqrt{(x+h)-4} - 3\sqrt{x-4}}{h}$$

$$= \lim_{h \to 0} \frac{3\left(\sqrt{(x+h)-4} - \sqrt{x-4}\right)\left(\sqrt{x+h-4} + \sqrt{x-4}\right)}{h\left(\sqrt{x+h-4} + \sqrt{x-4}\right)} = \lim_{h \to 0} \frac{3\left[(x+h-4) - (x-4)\right]}{h\left(\sqrt{x+h-4} + \sqrt{x-4}\right)}$$

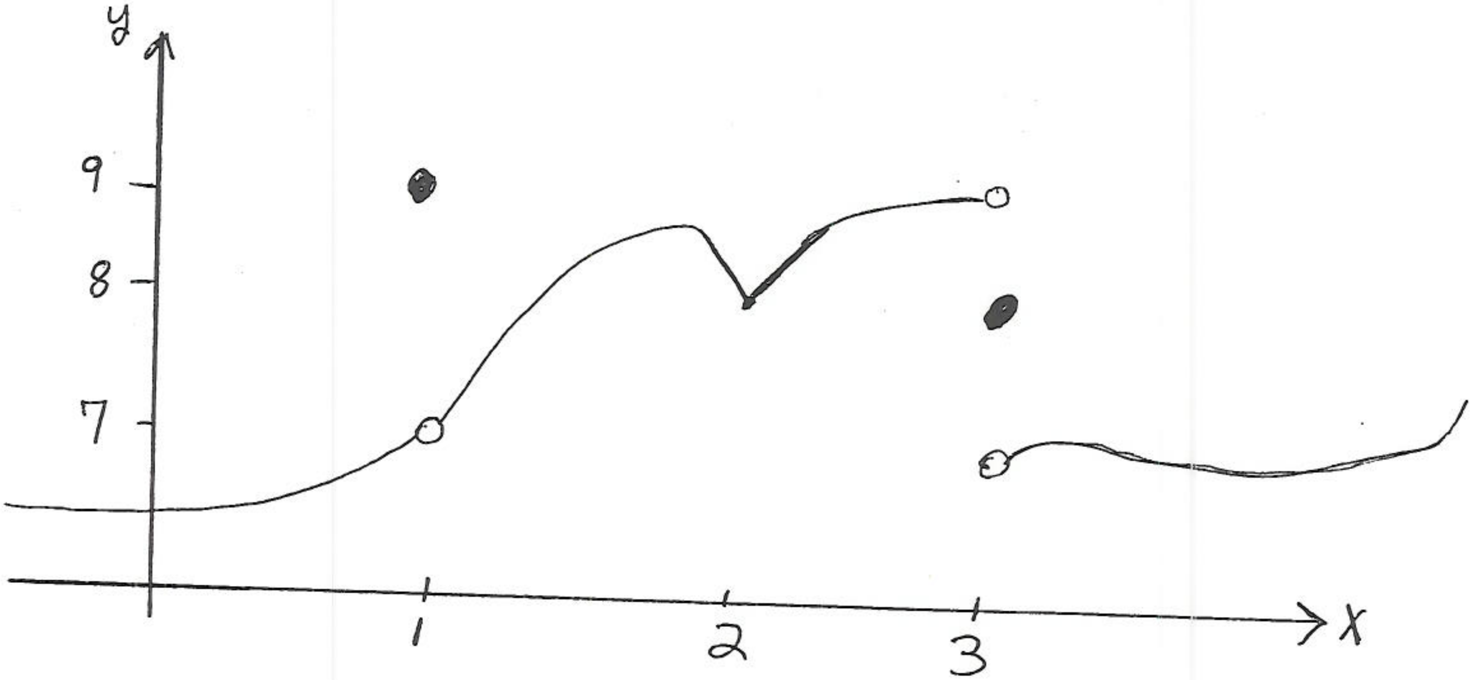
$$= \lim_{h \to 0} \frac{3(h)}{h\left(\sqrt{x+h-4} + \sqrt{x-4}\right)} = \lim_{h \to 0} \frac{3}{\sqrt{x+h-4} + \sqrt{x-4}} = \frac{3}{\sqrt{x-4} + \sqrt{x-4}}$$

$$= \frac{3}{2\sqrt{x-4}}$$

4. Graph $y = \sin 3x$. Mark a few points on each axis



5. **(The penalty for each mistake is four points.)** The picture represents the graph of $y = f(x)$.



(a) Fill in the blanks:

$f(1) = \underline{9}$ $\quad \lim_{x\to 1^+} f(x) = \underline{7}$ $\quad \lim_{x\to 1^-} f(x) = \underline{7}$ $\quad \lim_{x\to 1} f(x) = \underline{7}$

$f(2) = \underline{8}$ $\quad \lim_{x\to 2^+} f(x) = \underline{8}$ $\quad \lim_{x\to 2^-} f(x) = \underline{8}$ $\quad \lim_{x\to 2} f(x) = \underline{8}$

$f(3) = \underline{8}$ $\quad \lim_{x\to 3^+} f(x) = \underline{7}$ $\quad \lim_{x\to 3^-} f(x) = \underline{9}$ $\quad \lim_{x\to 3} f(x) = \underline{DNE}$

(b) Where is f continuous?

$f(x)$ is continuous for all X except $X=1$ and $x=3$.

(c) Where is f differentiable?

$f(x)$ is differentiable for all x except $x=1$, $x=2$, and $X=3$.

6. Find $\lim_{x\to 0} \frac{1-\cos x}{2x^2} = \lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{2x^2(1+\cos x)} = \lim_{x\to 0} \frac{1-\cos^2 x}{2x^2(1+\cos x)}$

$$= \lim_{x\to 0} \frac{\sin^2 x}{x^2} \frac{1}{2(1+\cos x)} = \left(\lim_{x\to 0} \frac{\sin x}{x}\right)^2 \lim_{x\to 0} \frac{1}{2(1+\cos x)} = \frac{1}{4}$$

7. Express $\cos(x-y)$ in terms of $\sin x$, $\sin y$, $\cos x$, and $\cos y$.

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

8. Compute the following limits:

(a) $\displaystyle \lim_{x\to 4^+} \frac{x+4}{x^2-16} = \lim_{x\to 4^+} \frac{x+4}{(x-4)(x+4)} = \lim_{x\to 4^+} \frac{1}{x-4} = +\infty$

(b) $\displaystyle \lim_{x\to 4^+} \frac{x^2-16}{x+4} = \lim_{x\to 4^+} \frac{(x+4)(x-4)}{x+4} = \lim_{x\to 4^+} x-4 = 0$

(c) $\displaystyle \lim_{x\to 4^+} \frac{x^2-16}{x-4} = \lim_{x\to 4^+} \frac{(x+4)(x-4)}{x-4} = \lim_{x\to 4^+} x+4 = 8$

(d) $\displaystyle \lim_{x\to 4^+} \frac{x-4}{x^2-16} = \lim_{x\to 4^+} \frac{(x-4)}{(x+4)(x-4)} = \lim_{x\to 4^+} \frac{1}{x+4} = \frac{1}{8}$