Math 141, Final Exam , 1999

PRINT Your Name:______________________________

There are 19 problems on 9 pages. One of the problems is worth 20 points. Each of the other problems is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** You might find the following formulas to be useful:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}.$$

1. Let $f(x) = \sqrt{3\sin^3 2x + \frac{3}{x}}$. Find $f'(x)$.

2. Compute $\int x\sqrt{x^2+1}dx$. (Be sure to check your answer.)

3. Find the area of the region which is bounded by $y = x^2$ and $y - x = 2$.

4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \frac{1}{3x}$.

5. Find $\displaystyle\lim_{x\to 2^+} \frac{x-2}{x^2-5x+6}$.

6. (20 points) Let

$$f(x) = \begin{cases} x & \text{if } x < 1, \\ 2 & \text{if } 1 \le x \le 2, \\ 4-x & \text{if } 2 < x. \end{cases}$$

   (a) Graph $y = f(x)$.
   (b) Fill in the blanks:

$f(1) =$ __ $\quad \lim_{x\to 1^+} f(x) =$ __ $\quad \lim_{x\to 1^-} f(x) =$ __ $\quad \lim_{x\to 1} f(x) =$ __

$f(2) =$ __ $\quad \lim_{x\to 2^+} f(x) =$ __ $\quad \lim_{x\to 2^-} f(x) =$ __ $\quad \lim_{x\to 2} f(x) =$ __

$f(3) =$ __ $\quad \lim_{x\to 3^+} f(x) =$ __ $\quad \lim_{x\to 3^-} f(x) =$ __ $\quad \lim_{x\to 3} f(x) =$ __

7. Find the legth of the curve $y = \frac{x^4}{4} + \frac{1}{8x^2}$ from $x = 1$ to $x = 2$.

8. Each edge of a cube is increasing at the rate of 4 inches per second. How fast is the surface area of the cube increasing when an edge is 12 inches long?

9. Let $f(x) = 1 + 8x^2 - x^4$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local maximum points, local minimum points, and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes of $y = f(x)$. Graph $y = f(x)$.

10. Graph $y = \sin\frac{x}{2}$. Mark a few points on each axis.

11. Find $\frac{dy}{dx}$ for $6x^4y^3 + \sin(2x^2y^3) = 19y^2$.

12. Consider the region A, which is bounded by the $x-$ axis, $y = x^3$, $x = 0$, and $x = 3$. Consider 150 rectangles, all with base $1/50$, which UNDER estimate the area of A . How much area is inside the 150 rectangles? (You must answer the question I asked. I expect an exact answer in closed form.)

13. Solve the Initial Value Problem $\frac{dy}{dx} = y^3$, $y(-1) = 1$. (Be sure to check your answer.)

14. Let R be the region between $y = x^2$ and the $x-$ axis, from $x = 1$ to $x = 2$. Find the volume of the solid which is obtained by revolving R about the $y-$ axis.

15. Define the definite integral $\displaystyle\int_a^b f(x)dx$.

16. Let R be the region between $y = x^2$ and the $x-$ axis, from $x = 1$ to $x = 2$. Find the volume of the solid which is obtained by revolving R about the $x-$ axis.

17. Farmer Brown has 100 feet of fence with which he plans to enclose a rectangular pen along one side of his 150-foot barn. (The side along the barn needs no fence.) What are the dimensions of the pen that has maximum area?

18. Compute $\displaystyle\int \frac{x+1}{\sqrt{x}} dx$. (Be sure to check your answer.)

19. A tank in the shape of a right circular cylinder, standing on its end, is full of water. The density of water is 62.4 pounds per cubic foot. If the height of the tank is 15 feet and the radius of the tank is 5 feet, then find the work done in pumping the water over the top edge of the tank.