Math 141, Exam 4, 1999

PRINT Your Name:____________________ Recitation Time ____________

There are 10 problems on 5 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!** You might find the following formulas to be useful:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{and} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}.$$

1. Consider the region bounded by $y = x^3$, the x-axis, $x = 0$ and $x = 1$. Revolve this region about the x-axis. What is the volume of the resulting solid?

2. Find the area of the region between $x = y^2$ and $2y + x = 3$.

3. Consider the region bounded by $y = x^3$, the x-axis, $x = 0$ and $x = 1$. Approximate the area of this region using 50 rectangles. The rectangles have equal bases. Make your recatangles OVER estimate the area of the region. How much area is INSIDE the 50 RECTANGLES? (Be sure to answer the question that I asked.)

4. Define the definite integral $\int_a^b f(x)dx$.

5. An open box with a capacity of 36 cubic feet is needed. If the box must be twice as long as it is wide, what dimensions would require the least amount of material?

6. Let $f(x) = x^4 - 2x^2$. Where is $f(x)$ increasing, decreasing, concave up, and concave down? What are the local extreme points and points of inflection of $y = f(x)$. Find all vertical and horizontal asymptotes. Graph $y = f(x)$.

7. Let $3x^4y^3 = \cos(5x^3y^6)$. Find $\frac{dy}{dx}$.

8. State both parts of the Fundamental Theorem of Calculus.

9. Solve $\frac{dy}{dx} = \frac{x}{y}$, with $y(1) = -2$. (Be sure to check your answer).

10. A 25-foot ladder is leaning against a wall. If the bottom of the ladder is pulled along the level pavement directly away from the wall at 3 feet per second, how fast is the top of the ladder moving down the wall when the foot of the ladder is 7 feet from the wall?