Math 141, Exam 2, 1999

PRINT Your Name:____________________ Recitation Time ____________

There are 10 problems on 4 pages. Each problem is worth 10 points. In one problem you are instructed to use the definition of the derivative; you MUST use the definition of the derivative in that problem. In the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **NO CALCULATORS!**

1. The volume of a cube is growing at the rate of 6 cubic inches per second. Find the rate of change of the cube's surface area at the instant when each side has length 10 inches.

2. **(The penalty for each mistake is five points.)** The picture represents the graph of $y = f(x)$. Fill in the blanks:

$$f(2) = \_\_ \qquad \lim_{x\to 2^+} f(x) = \_\_ \qquad \lim_{x\to 2^-} f(x) = \_\_ \qquad \lim_{x\to 2} f(x) = \_\_$$

$$f(3) = \_\_ \qquad \lim_{x\to 3^+} f(x) = \_\_ \qquad \lim_{x\to 3^-} f(x) = \_\_ \qquad \lim_{x\to 3} f(x) = \_\_$$

3. Find the maximum and the minimum of $f(x) = x^2 + 2x$ for $-2 \le x \le 1$.

4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \dfrac{1}{4x-3}$.

5. Graph $y = \cos 2x$. Mark a few points on each axis.

6. **(The penalty for each mistake is five points.)** Let

$$f(x) = \begin{cases} 4-x & \text{if } 2 \le x, \\ x & \text{if } 1 < x < 2, \text{ and} \\ 4-x & \text{if } x \le 1. \end{cases}$$

   (a) Graph $y = f(x)$.
   (b) Fill in the blanks:

$$f(0) = \_\_ \qquad \lim_{x\to 0^+} f(x) = \_\_ \qquad \lim_{x\to 0^-} f(x) = \_\_ \qquad \lim_{x\to 0} f(x) = \_\_$$

$$f(1) = \_\_ \qquad \lim_{x\to 1^+} f(x) = \_\_ \qquad \lim_{x\to 1^-} f(x) = \_\_ \qquad \lim_{x\to 1} f(x) = \_\_$$

$$f(2) = \_\_ \qquad \lim_{x\to 2^+} f(x) = \_\_ \qquad \lim_{x\to 2^-} f(x) = \_\_ \qquad \lim_{x\to 2} f(x) = \_\_$$

   (c) Where is $f(x)$ continuous?
   (d) Where is $f(x)$ differentiable?

7. Let $2x^3y^2 = \sin(2x^2y^4)$. Find $\frac{dy}{dx}$.

8. Find the equation of the line tangent to $f(x) = \sin^2 x$ at $x = \frac{\pi}{4}$.

9. Let $y = \frac{x}{\sin x}$. Find dy.

10. Let $y = \sqrt{x^3\cos^2(2x) + 19x^2}$. Find $\frac{dy}{dx}$.