Math 141, Exam 1, 1999

PRINT Your Name:____________________ Recitation Time ____________

There are 9 problems on 4 pages. Each problem, unless otherwise noted, is worth 10 points. In one problem you are instructed to use the definition of the derivative; you MUST use the definition of the derivative in that problem. In the other problems you may use any legitimate derivative rule. SHOW your work. $\boxed{CIRCLE}$ your answer. **NO CALCULATORS!**

1. Graph $y = \cos x - 2$.

2. Let $f(x) = 9x^4 + \frac{8}{x} + 3\sqrt{x} + 6$. Find $f'(x)$.

3. (14 points) **(The penalty for each mistake is four points.)** The picture represents the graph of $y = f(x)$.
   (a) Fill in the blanks:

$$f(1) = \_\_ \qquad \lim_{x\to1^+} f(x) = \_\_ \qquad \lim_{x\to1^-} f(x) = \_\_ \qquad \lim_{x\to1} f(x) = \_\_$$

$$f(2) = \_\_ \qquad \lim_{x\to2^+} f(x) = \_\_ \qquad \lim_{x\to2^-} f(x) = \_\_ \qquad \lim_{x\to2} f(x) = \_\_$$

$$f(3) = \_\_ \qquad \lim_{x\to3^+} f(x) = \_\_ \qquad \lim_{x\to3^-} f(x) = \_\_ \qquad \lim_{x\to3} f(x) = \_\_$$

   (b) Where is f continuous?
   (c) Where is f differentiable?

4. Use the DEFINITION of the DERIVATIVE to find the derivative of $f(x) = \frac{4}{x} - 3$.

5. Let $f(x) = (x+6)\sqrt{x}$. Find $f'(x)$.

6. Find the equation of the line tangent to $f(x) = 10x^{11} + 12x$ at $x = -1$.

7. Find $\lim_{x\to0} \frac{1-\cos x}{x^2}$.

8. (14 points) **(The penalty for each mistake is four points.)** Let

$$f(x) = \begin{cases} 2 - x^2 & \text{if } 1 \le x, \\ x + 1 & \text{if } 0 < x < 1, \text{ and} \\ -x & \text{if } x \le 0. \end{cases}$$

   (a) Graph $y = f(x)$.
   (b) Fill in the blanks:

$$f(0) = \_\_ \qquad \lim_{x\to0^+} f(x) = \_\_ \qquad \lim_{x\to0^-} f(x) = \_\_ \qquad \lim_{x\to0} f(x) = \_\_$$

$$f(1) = \_\_ \qquad \lim_{x\to1^+} f(x) = \_\_ \qquad \lim_{x\to1^-} f(x) = \_\_ \qquad \lim_{x\to1} f(x) = \_\_$$

$$f(2) = \_\_ \qquad \lim_{x\to2^+} f(x) = \_\_ \qquad \lim_{x\to2^-} f(x) = \_\_ \qquad \lim_{x\to2} f(x) = \_\_$$

   (c) Where is $f(x)$ continuous?
   (d) Where is $f(x)$ differentiable?

9. (12 points – 3 points for each part) Compute the following limits:

(a) $\displaystyle\lim_{x\to 3^+} \frac{x^2-x-6}{x-3}$

(c) $\displaystyle\lim_{x\to 3^+} \frac{x-3}{x^2-x-6}$

(c) $\displaystyle\lim_{x\to 3^+} \frac{x^2-x-6}{x+3}$

(d) $\displaystyle\lim_{x\to 3^+} \frac{x+3}{x^2-x-6}$