

**Math 141, Exam 1, 2000**

PRINT Your Name: \_\_\_\_\_ Section: \_\_\_\_\_

There are 8 problems on 4 pages. Problems 1 and 5 and are worth 15 points each. Problem 8 is worth 20 points. The other problems are worth 10 points each. In problem 3 you MUST use the definition of the derivative. SHOW your work.

*CIRCLE* your answer.

**NO CALCULATORS!**

1. **(The penalty for each mistake is four points.)** Let

$$f(x) = \begin{cases} x^2 + 2 & \text{if } x < 0, \\ 3 - x & \text{if } 0 \leq x \leq 1, \text{ and} \\ x + 1 & \text{if } 1 < x. \end{cases}$$

- (a) Graph  $y = f(x)$ .  
 (b) Fill in the blanks:

$$\begin{array}{cccc} f(0) = \_ & \lim_{x \rightarrow 0^+} f(x) = \_ & \lim_{x \rightarrow 0^-} f(x) = \_ & \lim_{x \rightarrow 0} f(x) = \_ \\ f(1) = \_ & \lim_{x \rightarrow 1^+} f(x) = \_ & \lim_{x \rightarrow 1^-} f(x) = \_ & \lim_{x \rightarrow 1} f(x) = \_ \\ f(2) = \_ & \lim_{x \rightarrow 2^+} f(x) = \_ & \lim_{x \rightarrow 2^-} f(x) = \_ & \lim_{x \rightarrow 2} f(x) = \_ \end{array}$$

- (c) Where is  $f(x)$  continuous?  
 (d) Where is  $f(x)$  differentiable?

2. Let  $f(x) = 3x - 1$  and  $g(x) = 2x^2 + 1$ .

- (a) Find  $(f \circ g)(x)$ .  
 (b) Find  $(g \circ f)(x)$ .

3. Use the DEFINITION of the DERIVATIVE to find the derivative of  $f(x) = 3\sqrt{x - 4}$ .

4. Graph  $y = \sin 3x$ . Mark a few points on each axis.

5. **(The penalty for each mistake is four points.)** The picture represents the graph of  $y = f(x)$ .

- (a) Fill in the blanks:

$$\begin{array}{cccc} f(1) = \_ & \lim_{x \rightarrow 1^+} f(x) = \_ & \lim_{x \rightarrow 1^-} f(x) = \_ & \lim_{x \rightarrow 1} f(x) = \_ \\ f(2) = \_ & \lim_{x \rightarrow 2^+} f(x) = \_ & \lim_{x \rightarrow 2^-} f(x) = \_ & \lim_{x \rightarrow 2} f(x) = \_ \\ f(3) = \_ & \lim_{x \rightarrow 3^+} f(x) = \_ & \lim_{x \rightarrow 3^-} f(x) = \_ & \lim_{x \rightarrow 3} f(x) = \_ \end{array}$$

- (b) Where is  $f(x)$  continuous?  
 (c) Where is  $f(x)$  differentiable?

6. Find  $\lim_{x \rightarrow 0} \frac{1 - \cos x}{2x^2}$ .

7. Express  $\cos(x - y)$  in terms of  $\sin x$ ,  $\sin y$ ,  $\cos x$ , and  $\cos y$ .

8. Compute the following limits:

(a)  $\lim_{x \rightarrow 4^+} \frac{x + 4}{x^2 - 16}$

(b)  $\lim_{x \rightarrow 4^+} \frac{x^2 - 16}{x + 4}$

(c)  $\lim_{x \rightarrow 4^+} \frac{x^2 - 16}{x - 4}$

(d)  $\lim_{x \rightarrow 4^+} \frac{x - 4}{x^2 - 16}$