A FRAMEWORK OF ROGERS-RAMANUJAN IDENTITIES AND THEIR ARITHMETIC PROPERTIES

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The two Rogers–Ramanujan q-series

$$\sum_{n=0}^{\infty} \frac{q^{n(n+\sigma)}}{(1-q)\cdots(1-q^n)},$$

where $\sigma = 0, 1$, play many roles in mathematics and physics. By the Rogers–Ramanujan identities, they are essentially modular functions. Their quotient, the Rogers–Ramanujan continued fraction, has the special property that its singular values are algebraic integral units. We find a framework which extends the Rogers–Ramanujan identities to doubly-infinite families of q-series identities. If $a \in \{1,2\}$ and $m, n \geq 1$, then we have

$$\sum_{\substack{\lambda \\ \lambda_1 \leq m}} q^{a|\lambda|} P_{2\lambda}(1, q, q^2, \ldots; q^n) = \text{“Infinite product modular function”}.$$

The $P_\lambda(x_1, x_2, \ldots; q)$ are extended Hall–Littlewood polynomials. We identify our q-series as specialized characters of affine Kac–Moody algebras, and show that their singular values are algebraic. Generalizing the Rogers–Ramanujan continued fraction, we prove in the case of $A_{2n}^{(2)}$ that the relevant q-series quotients are again algebraic integral units. This is joint work with Ken Ono and Ole Warnaar.

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