Math 141 **Calculus I** **2/27/2018**

- *Worksheet 4*

1. Use implicit differentiation to find $\frac{dy}{dx}$ in the following:

$$(a)\ 2xy + y^2 = x + y \qquad (b)\ x^3 = \frac{2x - y}{x + 3y}$$
$$(c)\ x^3 - xy + y^3 = 1 \qquad (d)\ xy = \cot(xy)$$
$$(e)\ (3xy + 7)^2 = 6y \qquad (f)\ x^4 + \sin(y) = x^3y^2$$
$$(g)\ y^2 = \frac{x - 1}{x + 1} \qquad (h)\ y \sin\left(\frac{1}{y}\right) = 1 - xy$$

2. For the given implicit equations, (a) verify that the given point lies on the curve, (b) find the tangent line to that point, and (c) find the line normal to the point (recall that if the tangent line has slope m, the normal will have slope $-1/m$).

$$(a)\ x^2 + xy - y^2 = 1, \quad (2, 3)$$
$$(b)\ x^2y^2 = 9, \quad (-1, 3)$$
$$(c)\ 2xy + \pi \sin(y) = 2\pi, \quad (1, \pi/2)$$

3. (a) Let $f(x) = x^3 - 3x^2 - 1$, with $x \geq 2$. Find the value of df^{-1}/dx at the point $x = -1 = f(3)$.

   (b) Let $f(x) = x^2 - 4x - 5$, with $x > 2$. Find the value of df^{-1}/dx at $x = 0 = f(5)$.

4. Find the following derivatives:

$$(a)\ y = \ln(3x) + x \qquad (b)\ y = \ln\left(\frac{3}{x}\right)$$
$$(c)\ y = \frac{1}{\ln(3x)} \qquad (d)\ y = \ln(\sin(x))$$
$$(e)\ y = \ln(t^2) \qquad (f)\ y = \big(\ln(x)\big)^3$$
$$(g)\ y = \ln(t^{3/2}) + \sqrt{t} \qquad (h)\ y = t\ln(\sqrt{t})$$
$$(i)\ y = \ln\left(\sqrt{\frac{1 + x}{1 - x}}\right) \qquad (j)\ y = \ln(\ln(\ln(x)))$$
$$(k)\ y = \ln(\sec(\ln(\theta))) \qquad (l)\ y = \sqrt{\ln(\sqrt{t})}$$

5. Use logarithmic differentiation to find the derivative of y with respect to the appropriate

variable:

$$(a)\ y = \sqrt{x(x+1)} \qquad (b)\ y = \sqrt{\frac{t}{t+1}}$$
$$(c)\ y = \sqrt{(x^2+1)(x-1)^2} \qquad (d)\ y = \frac{\theta+5}{\theta\cos(\theta)}$$
$$(e)\ y = \frac{1}{t(t+1)(t+2)} \qquad (f)\ y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$$

6. Compute the following derivatives:

$$(a)\ y = \cos^{-1}(x^2) \qquad (b)\ y = \cos^{-1}(1/x)$$
$$(c)\ y = \csc^{-1}(x^2+1) \qquad (d)\ y = \sin^{-1}(1-t)$$
$$(e)\ y = \sec^{-1}(2s+1) \qquad (f)\ y = \cos^{-1}(e^{-t})$$
$$(g)\ y = s\sqrt{1-s^2} + \cos^{-1}(s) \qquad (h)\ y = \ln(x^2+4) - x\tan^{-1}\left(\frac{x}{2}\right)$$

7. (a) Suppose that the radius r and the are $A = \pi r^2$ of a circle are differentiable with respect to t. Find an equation relating dA/dt and dr/dt.
   (b) Suppose that the radius r and surface area $S = 4\pi r^2$ of a sphere are differentiable functions of t. Find an equation relating dr/dt and dS/dt.

8. (a) Assume $y = 5x$ and $dx/dt = 2$. Find dy/dt.
   (b) If $y = x^2$ and $dx/dt = 3$, find dy/dt when $x = -1$.
   (c) If $x^2y^3 = 4/27$ and $dx/dt = 1/2$, then what is dy/dt when $x = 3$ and $y = 4$?

9. The radius r and height h of a right circular cone are related to the cone's volume V by the formula $V = \pi r^2 h$.
   (a) How is dV/dt related to dh/dt if r is constant?
   (b) How is dV/dt related to dr/dt if h is constant?
   (c) How is dV/dt related to both dr/dt and dh/dt, assuming neither h nor r is constant.

10. Let x and y be differentiable functions of t and let $s = \sqrt{x^2+y^2}$ be the distance between the points $(x, 0)$ and $(0, y)$.
    (a) How is ds/dt related to dx/dt if y is constant?
    (b) How is ds/dt related to dx/dt and dy/dt if neither x nor y is constant?
    (c) How is dx/dt related to dy/dt if s is constant?