Math 141 **Calculus I** **2/13/2018**

- *Worksheet 2*

1. For the following, find the equation for the tangent line to the curve at the given point.

$$(a)\ f(x) = 4 - x^2,\ (-1, 3)$$
$$(b)\ f(x) = 2\sqrt{x},\ (1, 2)$$
$$(c)\ f(x) = \frac{1}{x^2},\ (-1, 1)$$
$$(d)\ f(x) = \frac{x-1}{x+1},\ (0, -1)$$

2. For the following problems, recall that if height $s(t)$ is given as a function of time t, then the velocity at time t is $v(t) = \lim_{h \to 0} \frac{s(t+h) - s(t)}{h}$.

   (a) An object is dropped from a 100 meter tower. Its height after time t is given as $s(t) = 100 - 4.9t^2$. How fast is the object going after 2 seconds? How fast will the object be travelling right before it hits the ground?

   (b) A rocket is launched straight upward. Its height (in meters) as a function of time t (in seconds) is given by
   $$s(t) = -t^2 + 10t$$
   How fast is the rocket going after 2 seconds? Also, what is the maximum height that the rocket reaches?

   (c) What is the rate of change of the area of a circle with respect to the radius when $r = 3$?

3. Decide whether or not the following two functions have a well defined tangent line at the origin:

$$f(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

$$g(x) = \begin{cases} x \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

4. Using the definition of the derivative $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$, find the following

derivatives:

$$(a)\ f(x) = x^2 + 8, \qquad (b)\ g(t) = \frac{1}{t^2},$$
$$(c)\ r(s) = \sqrt{2s+1}, \qquad (d)\ F(x) = \frac{1}{\sqrt{x}},$$
$$(e)\ g(x) = \frac{x}{x-1}, \qquad (f)\ R(\theta) = \theta - \frac{1}{\theta},$$

5. Using *only the power rule*, find the following derivatives:

$$(a)\ f(x) = -x^2 + 3 \qquad (b)\ s(t) = 5t^3 - 3t^5$$
$$(c)\ w(z) = 3z^7 - 7z^3 + 21z^2 \qquad (d)\ f(x) = \sqrt[3]{x}$$
$$(e)\ s(t) = -2t^{-1} + \frac{4}{t^2} \qquad (f)\ r(s) = \frac{1}{3s^2} - \frac{5}{3s}$$
$$(g)\ r(\theta) = \frac{12}{\theta} - \frac{4}{\theta^3} + \frac{1}{\theta^4} \qquad (h)\ v(x) = \frac{1 + x - 4\sqrt{x}}{x}$$

6. Find the following derivatives using any derivative rules you prefer:

$$(a)\ y = (3 - x^2)\cdot(x^3 - x + 1) \qquad (b)\ w = (1 + x^2)(x^{3/4} - x^{-3})$$
$$(c)\ y = \frac{2x+5}{3x-2} \qquad (d)\ s = 2t^{3/2} + 3e^{2t}$$
$$(e)\ v = (1 - t)\cdot(1 + t^2)^{-1} \qquad (f)\ y = \frac{x^2 + 3e^x}{2e^x - x}$$
$$(g)\ r = e^{\theta}\cdot\left(\frac{1}{\theta^2} + \theta^{-\pi/2}\right) \qquad (h)\ w = \left(\frac{1+3z}{3z}\right)\cdot(3 - x)$$

7. The curve $y = ax^2 + bx + c$ passes through the point $(1, 2)$ and is tangent to the line $y = x$ at the origin. Find the values a, b, and c.

8. Find all points (x, y) on the graph $f(x) = 3x^2 - 4x$ with tangent lines parallel to the line $y = 8x + 5$.

9. Find the values a and b that make the following function differentiable at all points:

$$f(x) = \begin{cases} ax + b, & x > -1 \\ bx^2 - 3, & x \le -1 \end{cases}$$

10. The curves $y = x^2 + ax + b$ and $y = cx - x^3$ have a common tangent line at the point $(1, 0)$. Find a, b, and c.