- *Worksheet*

1. Find the average rate of change of the following functions on the given interval:

$$(a)\ f(x) = x^2 - 2x \ \textit{on}\ [-2, 4]$$
$$(b)\ h(t) = \cot(t) \ \textit{on}\ [\pi/6, \pi/2]$$
$$(c)\ R(\theta) = \sqrt{4\theta + 1} \ \textit{on}\ [0, 2]$$

2. Find the slope of the given equation at the given point:

$$(a)\ f(x) = x^2 - 5 \ \textit{at}\ x = 2$$
$$(b)\ f(x) = x^2 - 4x \ \textit{at}\ x = 1$$
$$(c)\ f(x) = x^3 - 3x^2 + 4 \ \textit{at}\ x = 2$$

3. For each part of the previous problem, find the equation of the tangent line to the given point.

4. Evaluate the following limits:

$$(a)\ \lim_{h\to 0} \frac{3}{\sqrt{3h+1}+1}$$
$$(b)\ \lim_{t\to 1} \frac{t^2 + t - 2}{t^2 - 1}$$
$$(c)\ \lim_{x\to 2} \frac{x^2 - 7x + 10}{x - 2}$$
$$(d)\ \lim_{x\to 1} \frac{x^{-1} - 1}{x - 1}$$
$$(e)\ \lim_{v\to 2} \frac{v^3 - 8}{v^4 - 16}$$

5. If $2 - x^2 \leq g(x) \leq 2\cos(x)$ for all x, find $\lim_{x\to 0} g(x)$.

6. Let $\epsilon > 0$. For the given values of δ, verify that the following limits satisfy the precise

definition of the limit:

$$(a)\ \lim_{x\to 2} x+1=3,\ \delta=\epsilon$$
$$(b)\ \lim_{x\to 3} 2x-2=4,\ \delta=\epsilon/2$$
$$(c)\ \lim_{x\to 3} \sqrt{x+1}=2,\ \delta=\epsilon$$
$$(d)\ \lim_{x\to 0} \sin(x)=0,\ \delta=\epsilon$$
$$(e)\ \lim_{x\to 2} 1/x=1/2,\ \delta=\min\{1,\epsilon/2\}$$
$$(f)\ \lim_{x\to 3} 3-7x=-18,\ \delta=\epsilon/7$$

7. For what values of a are the following functions continuous:

$$f(x)=\begin{cases} a^2x-2a,\ x\geq 2 \\ 12,\ x<2 \end{cases}$$

$$f(x)=\begin{cases} \frac{x-a}{a+1},\ x<0 \\ x^2+a,\ x\geq 0 \end{cases}$$

8. Show that the function $f(x)=x^3-3x-1$ has at least 2 roots in the interval $[-2,0]$.