§7.1.

P3. Solution:

$$\mathcal{L}\{e^{3t+1}\} = \int_0^{+\infty} e^{-st} e^{3t+1} dt$$
$$= e\int_0^{+\infty} e^{-(s-3)t} dt$$
$$= e\left[-\frac{1}{s-3} e^{-(s-3)t}\right]_{t=0}^{+\infty}$$
$$= e\cdot\left[\frac{1}{s-3}\right]$$
$$= \frac{e}{s-3} \quad \text{for all } s>3.$$

P4. Solution:

$$\mathcal{L}\{\cos t\} = \int_0^{+\infty} e^{-st}\cos t\, dt$$
$$= \underbrace{\left[e^{-st}\sin t\right]_{t=0}^{+\infty}}_{=0} + s\int e^{-st}\sin t\, dt.$$
$$= s\Big(\left[-e^{-st}\cos t\right]_{t=0}^{+\infty} - s\underbrace{\int_0^{+\infty} e^{-st}\cos t\, dt}_{\mathcal{L}\{\cos t\}}\Big)$$
$$= s(1-s\mathcal{L}\{\cos t\})$$
$$= s-s^2\mathcal{L}\{\cos t\}$$
$$\Rightarrow (1+s^2)\mathcal{L}\{\cos t\} = s$$
$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1} \quad \text{for } s>0.$$

P6. Solution:

$$\mathcal{L}\{\sin^2 t\} = \int_0^{+\infty} e^{-st}\sin^2 t\, dt$$
$$= \int_0^{+\infty} e^{-st}\left(\frac{1-\cos 2t}{2}\right) dt.$$
$$= \frac{1}{2}\Big(\underline{\int_0^{+\infty} e^{-st} dt} - \underline{\int_0^{+\infty} e^{-st}\cos 2t\, dt}\Big)$$

$$\int_0^{+\infty} e^{-st} dt = \frac{1}{s}$$
$$\int_0^{+\infty} e^{-st}\cos 2t\, dt = \frac{s}{s^2+4}$$
$$\Rightarrow \mathcal{L}\{\sin^2 t\} = \frac{1}{2}\left[\frac{1}{s}-\frac{s}{s^2+4}\right] \quad \text{for } s>0.$$

P8. Solution:

$$f(t)=\begin{cases} 0 & 0\le t\le 1 \\ 1 & 1<t\le 2 \\ 0 & t>2. \end{cases}$$

thus,

$$\mathcal{L}\{f(t)\} = \int_0^{+\infty} e^{-st} f(t)\, dt$$
$$= \int_0^1 e^{-st}\cdot 0\, dt + \int_1^2 e^{-st}\cdot 1\, dt + \int_2^{+\infty} e^{-st}\cdot 0\, dt$$
$$= \int_1^2 e^{-st} dt$$
$$= \left[-\frac{1}{s} e^{-st}\right]_{t=1}^{2}$$
$$= -\frac{1}{s}(e^{-2s}-e^{-s}) \quad \text{for } s>0.$$

P11. Solution:

$$\mathcal{L}\{\sqrt{t}+3t\} = \mathcal{L}\{\sqrt{t}\}+3\mathcal{L}\{t\}$$
$$= \frac{\Gamma(\frac{3}{2})}{s^{3/2}} + 3\frac{\Gamma(2)}{s^2} \qquad \left(\Gamma(\tfrac{3}{2})=\tfrac{1}{2}\sqrt{\pi}\right)$$
$$= \frac{\frac{1}{2}\sqrt{\pi}}{s^{3/2}} + \frac{3}{s^2}$$
$$= \frac{1}{2}\sqrt{\frac{\pi}{s^3}} + \frac{3}{s^2} \quad \text{for } s>0$$

P14. Solution:

$$\mathcal{L}\{t^{3/2}-e^{-10t}\} = \mathcal{L}\{t^{3/2}\}-\mathcal{L}\{e^{-10t}\}.$$
$$= \frac{\Gamma(5/2)}{s^{5/2}} - \frac{1}{s+10}$$
$$= \frac{\frac{3}{4}\sqrt{\pi}}{s^{5/2}} - \frac{1}{s+10}$$
$$= \frac{3}{4}\sqrt{\frac{\pi}{s^5}} - \frac{1}{s+10} \quad \text{for } s>0$$

P15. Solution:

$$\mathcal{L}\{1+\cosh 5t\} = \mathcal{L}\{1\}+\mathcal{L}\{\cosh 5t\}$$
$$= \frac{1}{s}+\frac{s}{s^2-25} \quad \text{for } s>5$$

P.18. Solution:

$$\mathcal{L}\{\sin 3t\cos 3t\} = \mathcal{L}\{\tfrac{1}{2}\sin 6t\}.$$
$$= \frac{1}{2}\mathcal{L}\{\sin 6t\}$$
$$= \frac{1}{2}\cdot\frac{6}{s^2+36} \quad \text{for } s>0.$$
$$= \frac{3}{s^2+36}$$

P19. Solution:

$$\mathcal{L}\{(1+t)^3\} = \mathcal{L}\{1+3t+3t^2+t^3\}$$
$$= \mathcal{L}\{1\}+3\mathcal{L}\{t\}+3\mathcal{L}\{t^2\}+\mathcal{L}\{t^3\}$$
$$= \frac{1}{s} + 3\cdot\frac{2!}{s^2} + 3\cdot\frac{2!}{s^3} + \frac{3!}{s^4}$$
$$= \frac{1}{s} + \frac{3}{s^2} + \frac{6}{s^3} + \frac{6}{s^4}$$

P20. Solution:

$$\mathcal{L}\{te^t\} = \int_0^{+\infty} e^{-st}\, t e^t\, dt$$
$$= \int_0^{+\infty} e^{-(s-1)t}\, t\, dt$$
$$= \underbrace{\left[-\frac{1}{s-1} e^{-(s-1)t}\, t\right]_{t=0}^{+\infty}}_{0} + \int_0^{+\infty} \frac{1}{s-1} e^{-(s-1)t}\, dt$$
$$= \frac{1}{s-1}\int e^{-(s-1)t}\, dt$$
$$= \frac{1}{s-1}\int e^{-st} e^t\, dt$$
$$= \frac{1}{s-1}\mathcal{L}\{e^t\}$$
$$= \left(\frac{1}{s-1}\right)\cdot\left(\frac{1}{s-1}\right)$$
$$= \frac{1}{(s-1)^2} \quad \text{for } s>1.$$

P24. Solution:

$$\mathcal{L}^{-1}\{s^{-3/2}\} = \frac{2}{\sqrt{\pi}}\mathcal{L}^{-1}\left\{\frac{\sqrt{\pi}}{2} s^{-3/2}\right\}$$
$$= \frac{2}{\sqrt{\pi}} t^{\frac{1}{2}}$$

P26. Solution:

$$\mathcal{L}^{-1}\left\{\frac{1}{s+5}\right\} = e^{-5t}$$

P28. Solution:

$$\mathcal{L}^{-1}\left\{\frac{3s+1}{s^2+4}\right\} = 3\mathcal{L}^{-1}\left\{\frac{s}{s^2+\underbrace{4}_{2^2}}\right\} + \mathcal{L}^{-1}\left\{\frac{1}{s^2+\underbrace{4}_{2^2}}\right\}$$
$$= 3\cos 2t + \frac{1}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2+4}\right\}$$
$$= 3\cos 2t + \frac{1}{2}\sin 2t$$

P30. Solution:

$$\mathcal{L}^{-1}\left\{\frac{9+s}{4-s^2}\right\} = \mathcal{L}^{-1}\left\{\frac{-s-9}{s^2-4}\right\}.$$
$$= -\mathcal{L}^{-1}\left\{\frac{s}{s^2-2^2}\right\} - \mathcal{L}^{-1}\left\{\frac{9}{s^2-2^2}\right\}$$
$$= -\cosh 2t - \frac{9}{2}\mathcal{L}^{-1}\left\{\frac{2}{s^2-2^2}\right\}.$$
$$= -\cosh 2t - \frac{9}{2}\sinh 2t$$