

§4.1.

P1. Solution:

$$\begin{cases} X_1' = X_2 \\ X_2' = -7X_1 - 3X_2 + t^2 \end{cases}$$

P6 Solution:

$$\begin{cases} X_1' = X_2 \\ X_2' = X_3 \\ X_3' = X_4 \\ X_4' = -X_1 + 3X_2 - 6X_3 + 6\sin t \end{cases}$$

P8. Solution:

$$\begin{cases} X_1' = X_2 \\ X_2' = X_3 \\ X_3' = -\frac{5}{t^3}X_1 - \frac{3}{t^2}X_2 + \frac{2}{t}X_3 + \frac{\ln t}{t^3} \end{cases}$$

P11. Solution:

$$\begin{aligned} X_1 = X, X_2 = X', X_3 = Y, X_4 = Y' \\ \begin{cases} X_1' = X_2 \\ X_2' = -\frac{kX_1}{(X_1^2 + X_3^2)^{3/2}} \\ X_3' = X_4 \\ X_4' = -\frac{kX_3}{(X_1^2 + X_3^2)^{3/2}} \end{cases} \end{aligned}$$

P15 Solution:

$$\begin{aligned} X_1 = X, X_2 = X', X_3 = Y, X_4 = Y' \\ X_5 = Z, X_6 = Z' \\ \begin{cases} X_1' = X_2 \\ X_2' = 3X_1 - X_3 + 2X_5 \\ X_3' = X_4 \\ X_4' = X_1 + X_3 - 4X_5 \\ X_5' = X_6 \\ X_6' = 5X_1 - X_3 - X_5 \end{cases} \end{aligned}$$

P17. Solution:

$$X'' = Y' = -X \Rightarrow X'' + X = 0$$

$$\text{Characteristic equation: } Y^2 + 1 = 0 \\ Y = \pm i$$

$$\Rightarrow X(t) = A\cos t + B\sin t \Rightarrow Y(t) = -A\sin t + B\cos t$$

P19. Solution:

$$X'' = -2Y' = -2 \cdot (2X) = 4X \Rightarrow$$

$$X'' + 4X = 0$$

$$\text{Characteristic equation: } Y^2 + 4 = 0 \\ Y = \pm 2i$$

$$\Rightarrow X(t) = A\cos 2t + B\sin 2t$$

$$X(0) = 1 \Rightarrow A = 1 \Rightarrow X(t) = \cos 2t + B\sin 2t$$

$$Y(t) = \frac{X'}{(-2)} = -\frac{1}{2}(-2\sin 2t + 2B\cos 2t) \\ = \sin 2t - B\cos 2t$$

$$Y(0) = 0 \Rightarrow -B = 0, \text{ Thus}$$

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$$

P22. Solution:

$$X'' = Y' = 6X - Y = 6X - X'$$

$$\Rightarrow X'' + X' - 6X = 0$$

$$\text{Characteristic equation: } Y^2 + Y - 6 = 0 \\ (Y+3)(Y-2) = 0$$

$$Y_1 = -3, Y_2 = 2$$

$$\Rightarrow X(t) = Ae^{-3t} + Be^{2t}$$

$$\Rightarrow Y(t) = X'(t) = -3Ae^{-3t} + 2Be^{2t}$$

$$X(0) = 1 \Rightarrow A + B = 1$$

$$Y(0) = 2 \Rightarrow -3A + 2B = 2 \Rightarrow \begin{cases} A = 0 \\ B = 1 \end{cases}$$

Thus

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} e^{2t} \\ 2e^{2t} \end{pmatrix}$$

P26. solution:

$$x'' = y' = -9x + 6y = -9x + 6x'$$

$$\Rightarrow x'' - 6x' + 9x = 0$$

characteristic equation: $r^2 - 6r + 9 = 0$

$$(r-3)^2 = 0$$

$$r_1 = r_2 = 3$$

$$\Rightarrow x(t) = e^{3t}(At + B)$$

$$\begin{aligned} \Rightarrow y(t) = x'(t) &= 3e^{3t}(At + B) + Ae^{3t} \\ &= e^{3t}(3At + 3B + A) \end{aligned}$$