

§24

P1. Solution:

$$y' = -y, y(0) = 2; y(x) = 2e^{-x} \quad (y(\frac{1}{2}) \approx 1.2131)$$

i) $h = 0.25, x_0 = 0, y_0 = 2, f(x, y) = -y.$

$$x_1 = x_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + (-y_0) \cdot h$$

$$= 2 + (-2) \cdot 0.25 = 1.5$$

$$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h$$

$$= 1.5 + (-1.5) \cdot 0.25 = \underline{1.125}$$

$$|y_2 - y(\frac{1}{2})| \approx \underline{0.0881}$$

ii) $h = 0.1$

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + (-y_0) \cdot h$$

$$= 2 + (-2) \cdot (0.1) = 1.9$$

$$x_2 = x_1 + h = 0.2$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h$$

$$= 1.9 + (-1.9) \cdot (0.1) = 1.62$$

$$x_3 = x_2 + h = 0.3$$

$$y_3 = y_2 + f(x_2, y_2) \cdot h = 1.4580$$

$$x_4 = x_3 + h = 0.4$$

$$y_4 = y_3 + f(x_3, y_3) \cdot h = 1.3122$$

$$x_5 = x_4 + h = 0.5$$

$$y_5 = y_4 + f(x_4, y_4) \cdot h \approx 1.1810$$

$$|y_5 - y(\frac{1}{2})| \approx \underline{0.0}$$

P4. Solution:

$$y' = x - y, y(0) = 1; y(x) = 2e^{-x} + x - 1$$

$$(y(\frac{1}{2}) \approx 0.7131)$$

$$x_0 = 0, y_0 = 1, f(x, y) = x - y$$

i) $h = 0.25$

$$x_1 = x_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + (x_0 - y_0) \cdot h$$

$$= 1 + (0 - 1) \cdot (0.25) = 0.75$$

$$x_2 = x_1 + h = 0.5$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h$$

$$= 0.75 + (0.25 - 0.75) \cdot (0.25) = \underline{0.625}$$

$$|y_2 - y(\frac{1}{2})| \approx \underline{0.0881}$$

ii) $h = 0.1$

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = 1 + (0 - 1) \cdot (0.1) = 0.9$$

$$x_2 = x_1 + h = 0.2$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h = 0.9 + (0.1 - 0.9) \cdot 0.1$$

$$= 0.82$$

$$x_3 = x_2 + h = 0.3$$

$$y_3 = y_2 + f(x_2, y_2) \cdot h = 0.758$$

$$x_4 = x_3 + h = 0.4$$

$$y_4 = y_3 + f(x_3, y_3) \cdot h = 0.7122$$

$$x_5 = x_4 + h = 0.5$$

$$y_5 = y_4 + f(x_4, y_4) \cdot h \approx 0.6810$$

$$|y_5 - y(\frac{1}{2})| \approx \underline{0.0321}$$

P6. Solution:

$$y' = -2xy, y(0) = 2; y(x) = 2e^{-x^2} \quad (y(\frac{1}{2}) \approx 1.5576)$$

$$x_0 = 0, y_0 = 2, f(x, y) = -2xy.$$

i) $h = 0.25$

$$x_1 = x_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + 2 \cdot x_0 \cdot y_0 \cdot h$$

$$= 2 + 2 \cdot 0 \cdot 2 \cdot (0.25) = 2$$

$$x_2 = x_1 + h = 0.5$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h = 2 + 2 \cdot (0.25) \cdot 2 \cdot (0.25)$$

$$= 1.75$$

$$|y_2 - y(\frac{1}{2})| \approx \underline{0.1924}$$

ii) $h = 0.1$

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + 2 \cdot x_0 \cdot y_0 \cdot h$$

$$= 2 + 2 \cdot 0 \cdot 2 \cdot 0.1 = 2$$

$$x_2 = x_1 + h = 0.2$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h = 2 - 2 \cdot (0.1) \cdot 2 \cdot (0.1) = 1.96$$

$$x_3 = x_2 + h = 0.3$$

$$y_3 = y_2 + f(x_2, y_2) \cdot h = 1.8816$$

$$x_4 = x_3 + h = 0.4$$

$$y_4 = y_3 + f(x_3, y_3) \cdot h \approx 1.7687$$

$$x_5 = x_4 + h = 0.5$$

$$y_5 = y_4 + f(x_4, y_4) \cdot h \approx 1.6272$$

$$|y_5 - y(\frac{1}{2})| \approx 0.0696$$

$$x_5 = x_4 + h = 0.5$$

$$y_5 = y_4 + f(x_4, y_4) \cdot h \approx 0.498$$

$$|y_5 - y(\frac{1}{2})| \approx 0.0143$$

P8. solution:

$$y' = e^{-y}, y(0) = 0; \psi(x) = \ln(x+1), (y(\frac{1}{2}) \approx 0.4055)$$

$$x_0 = 0, y_0 = 0, f(x, y) = e^{-y}$$

i) $h = 0.25$

$$x_1 = x_0 + h = 0 + 0.25 = 0.25$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + e^{-y_0} \cdot h \\ = 0 + e^{-0} \cdot (0.25) = 0.25$$

$$x_2 = 0.5$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h \approx 0.4447$$

$$|y_2 - y(\frac{1}{2})| \approx 0.0392$$

ii) $h = 0.1$

$$x_1 = x_0 + h = 0 + 0.1 = 0.1$$

$$y_1 = y_0 + f(x_0, y_0) \cdot h = y_0 + e^{-y_0} \cdot h \\ = 0 + e^{-0} \cdot 0.1 = 0.1$$

$$x_2 = x_1 + h = 0.1 + 0.1 = 0.2$$

$$y_2 = y_1 + f(x_1, y_1) \cdot h \\ = 0.1 + e^{-0.1} \cdot (0.1) \approx 0.1905$$

$$x_3 = x_2 + h = 0.3$$

$$y_3 = y_2 + f(x_2, y_2) \cdot h \approx 0.2731$$

$$x_4 = x_3 + h = 0.4$$

$$y_4 = y_3 + f(x_3, y_3) \cdot h \approx 0.3492$$